DUAL STRUCTURES OF PROBABILISTIC PSEUDOMETRIC SPACES IN RELATION WITH FUZZY INDISTINGUISHABILITIES[1]

F. Prats(*) - R. Rovira(**) - M. Sánchez(*)

The transitivity property of classical equivalence relations is not satisfied in most of the problems coming from Soft Sciences. In 1901, H. Poincaré, established the idea that equality is transitive only in a strictly mathematical frame. Moreover, in the real life, "equal" means "indistinguishable" and indistinguishability is in general not transitive, and so not an equivalence relation. In order to solve this problem, the classical concept of equivalence relation was modified by K. Menger [3], L.A. Zadeh [10] and E. Ruspini [5], who defined three reflexive and symmetrical fuzzy relations using different t-norms in order to modelize the transitivity property. In [8] a unified point of view of these three definitions is given: the concept of fuzzy indistinguishability operator associated with a t-norm T.

In the same way that Fuzzy indistinguishability operators, with values in [0,1], appeared from classical equivalence relations, with values in $\{0,1\}$, the probabilistic indistinguishability operators, that is to say, with values in the space Δ^+ of probability distribution functions of positive random variables, arise now in a natural way [4].

The relation between fuzzy indistinguishabilities and some generalized metrics suggests the definition of a kind of probabilistic indistinguishabilities related, in a similar way, to probabilistic metric spaces [7]. In probabilistic metric spaces the triangle inequality is expressed by means of a triangle function; in our work, in the definition of their dual structures, what allows us to obtain a minimum level of transitivity are the triangle cofunctions [6]. Given a strong negation N in Δ^+ and a triangle function τ, if ζ is the dual triangle cofunction of τ via the negation N, every ζ-indistinguishability has in a unique way associated a probabilistic pseudometric space structure under τ, and conversely.

The introduction of a set of observers permit us to link up the fuzzy indistinguishability relations with the probabilistic indistinguishability relations. We see also in this work that a particular case of probabilistic indistinguisha-

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... interpreted as fuzzy ones.

... to interpret the ζ-transitivity and the ζ- indistinguishabilities in ... probabilistic truth-values, we define an operation that modelizes ... "if $p(x,y)$ and $p(y,z)$, then $p(x,z)$".

... we obtain a representation theorem that establishes an effective ... construct examples of ζ-indistinguishabilities on X by proving that ... generated by a family of probabilistic subsets of X. This method ... representation theorem for probabilistic pseudometric spaces, due ... unity between these two concepts.

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... SPAIN

(**) Dep. Mètodes Quantitatius
E.S.A.D.E.
Av. Pedralbes, 60
08034 Barcelona. SPAIN.