ON PSEUDO-TRANSITIVE FUZZY RELATIONS

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We consider a generalization of the well-known notion of fuzzy transitive relations which can be used to determine lower and upper transitive approximations of a given fuzzy relation. This is of importance in decision-making and clustering where the frequently used transitive closure may fail.

Denotations: $\mathcal{X}$-classical set (basic space); $\mathcal{F}$ - set of all fuzzy relations over $\mathcal{X}\times\mathcal{X}$ with values in [0,1]; "$\circ$"-fuzzy composition with continuous t-norm.

Definition 1: Let $R, A \in \mathcal{F}$. We call R pseudo-transitive (pt) with respect to A iff

$$R^2 \wedge A \leq R \tag{1}$$

("$\wedge$" means pointwise minimum). The set of all pt-relations is denoted by $\mathcal{F}_A$.

Definition 2: Let $R, A \in \mathcal{F}$. $R_U \in \mathcal{F}_A$ is an upper pt-relation (uptr) iff

$$R_U \geq R$$

holds (for lower ptr R_L (lptr) correspondingly).

Now let $T_1 = R$, $T_i = (R \circ T_{i-1} \vee T_{i-1} \circ R) \wedge A$, $i = 1, 2, \ldots$ (2)

("$\vee$" for pointwise maximum).

Theorem: The set of uptr has a least element given by

$$\hat{R} = \bigvee_{i \geq 1} T_i \,. \tag{3}$$

For lptr we can show the existence of maximal elements using Zorn's Lemma. Their determination is an open problem. Nevertheless, using the special form

$$R_L = T \wedge R \tag{4}$$

with T transitive. Then we get

$$\begin{aligned}(T \wedge R)^2 \wedge A &\leq (T^2 \wedge T \circ R \wedge R \circ T \wedge R^2) \wedge A \\ &\leq (T \wedge T \circ R \wedge R \circ T \wedge R^2) \wedge A \end{aligned} \tag{5}$$

and we have to determine T such that

$$(T \wedge R)^2 \wedge A \leq T \wedge R \ .$$

From (5) follows that it is sufficient to demand

$$T \circ R \leq U$$

or

$$T \circ R \leq R$$

where U is a known relation $\geq R$.

Particularly, using (3) and (4) one gets transitive inclusions for a given fuzzy (maybe non-transitive) relation.

References

1. D. Dubois and H. Prade, Fuzzy Sets and Systems: Theory and Applications. Academic Press, N.Y. 1980.

2. M. Wagenknecht, On pseudo-transitive approximations of fuzzy relations. Submitted to FSS.

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