FUZZY IMPLICATION RELATION

- definition and basic properties -

C. Gherasim, H.N. Teodorescu

Extended abstract

The fuzzy implication operator, also named ply operator, plays an important role in approximate reasoning. Consequently, many authors paid attention to it. As a result at least seventy-two relations for the fuzzy implication operator (1) and four rules of fuzzy inference (4) exist. This is a wide range of relations and rules but the choice of the couple (implication operator; rule of inference) to model a certain real system is a problem. Z. CAO and A. Kandel (1) found just five of seventy-two operators having good robustness in all the cases they considered. Other authors select eleven (2) or ten (4) operators most significant in applications. Despite of the numerous definitions for the fuzzy implication operator, there are many cases of practical interest not very well modelled - whatever the couple (implication operator ; rule of inference) would be. Thus we are motivated to introduce a new relation for the fuzzy implication operator definition.

Let $\tilde{x}$ and $\tilde{y}$ be two linguistic variables and let u and v be their measures; $u \in U$, $v \in V$. The linguistic degrees of the variables x and y are given through fuzzy sets $\tilde{A}_i(\tilde{x})$, $\tilde{B}_j(\tilde{y})$ defined by membership functions

$$\mu_{\tilde{A}_i}(u): U \to \mathbf{L} \quad \text{and} \quad \mu_{\tilde{B}_j}(v): V \to \mathbf{L} \quad \text{where } i \in I,\ j \in J \quad \text{with}$$

I and J being two families of indices and $\mathbf{L} = (L, \wedge, \vee)$ is a lattice with $0,1 \in L$. Further on we consider the membership functions as convex functions (not a restrictive assumption for applications). The fuzzy implication operator, linguistically expressed by the statement "If $\tilde{x}$ is $\tilde{A}_i$ then $\tilde{y}$ is $\tilde{B}_j$", is generally defined as

$$\mu_{\tilde{A}_i \to \tilde{B}_j}(u,v) = \mu_{\tilde{A}_i}(u) \circ \mu_{\tilde{B}_j}(v) \quad \text{where} \quad \mu_{\tilde{A}_i \to \tilde{B}_j}(u,v)$$

denotes the membership function defining the set $\tilde{A}_i \to \tilde{B}_j$; we shall use a shorter notation for this function, namely Imp. The result of a modus ponens inference rule for a set $\tilde{A}_p$ in the antecedent is a set $\tilde{B}_q$ in the consequent obtained by

$$\tilde{B}_q = \mathrm{Proj}_V(\tilde{A}_p \wedge \mathrm{Imp}(\tilde{A}_i, \tilde{B}_j))$$

Due to the non-monotonic character of the membership functions in the antecedent and in the consequent, the resulting set can be the same for two different sets in the antecedent and this is confusing in many application, especially in control. Knowing the cause the ambiguity can be eliminated by spliting the membership functions into two momotonic parts and using them separately.

The monotonic parts would be: $\mu_{\widetilde{A}_i}(u) = \mu'_{\widetilde{A}_i}(u) + \mu''_{\widetilde{A}_i}(u)$ defined as:

$$\mu'_{\widetilde{A}_i}(u) = \begin{cases} \mu_{\widetilde{A}_i}(u) & \text{for } u < u_m \\ 0 & \text{otherwise} \end{cases} \qquad \mu''_{\widetilde{A}_i}(u) = \begin{cases} \mu_{\widetilde{A}_i}(u) & \text{for } u > u_m \\ 0 & \text{otherwise} \end{cases}$$

The value u_m determines the two monotonic functions; u_m always exists because of the convex character of the membership functions we assumed. To use these monotonic parts we need a pair of relations OR selectable. Hereby we propose a general definition of the form:

$$\mathrm{Imp}^*(\widetilde{A}_i,\widetilde{B}_j) = a_1 \cdot \mathrm{Imp}_1^*(\widetilde{A}_i,\widetilde{B}_j) \vee a_2 \cdot \mathrm{Imp}_2^*(\widetilde{A}_i,\widetilde{B}_j)$$

where Imp_1^* and Imp_2^* are fuzzy implication relations OR selectable making the pair of so called $*$-implications. a_1 and a_2 are coefficients selecting between those two $*$-implications, hereby named "selecting coefficients".

Especially for control problems the following pair of $*$-implications was found to be very useful:

$$\mathrm{Imp}_1^*(\widetilde{A}_i,\widetilde{B}_j) = \mu_{\widetilde{B}_j}(v) \wedge \mu_{\widetilde{B}_j}(v+\Delta v) \qquad \text{where}$$

$$\mathrm{Imp}_2^*(\widetilde{A}_i,\widetilde{B}_j) = \mu_{\widetilde{B}_j}(v) \wedge \mu_{\widetilde{B}_j}(v-\Delta v) \qquad \Delta v = v_2 - v_1 \quad \text{and}$$

$$v_1 = \sup \mu'^{-1}_{\widetilde{B}_j}(\mathrm{Poss}(\widetilde{A}_i/\widetilde{A}_p))$$

$$v_2 = \sup \mu''^{-1}_{\widetilde{B}_j}(\mathrm{Poss}(\widetilde{A}_i/\widetilde{A}_p))$$

Using these relations in a modus ponens inference rule for two different sets in the antecedent, two different sets as the consequent will appear.

The relations above lead to a finer action in control problems, to a better noise and membership functions shapes immunity, to the errors diminishment.

References

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Mail: C. Gherasim, str. Păcurari nr.68, Iaşi, RO-6600