# SOME THEORETICAL BASIS FOR CLUSTER ANALYSIS BASED ON FUZZY TRANSITIVE RELATION[1]

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Since Zadeh's paper "Similarity relations and fuzzy orderings" [9], several attempts to extend the concept of equivalence relation and its associated partition, to the fuzzy framework have been made.

Two main trends have been followed. The first one, defining fuzzy partition and studying later the properties of the relations associated to them [1,6]. The second one [5,7,8], which will be used in this paper, stablishes the conditions that a fuzzy partition must fulfil in order to define, univocally, a T-indistinguishability operator [8].

This last definition links up the metric defined in the basic set $X$ with a S-metric in the unit interval.

In fact, this approach is a generalization of the one given for classical clusters in terms of its characteristic functions, where the underlying metric is the discrete distance in the two-point set $\{0,1\}$.

Following our paper "on m-fuzzy cluster coverages" [3], we introduce, in a natural way, the concept of classification associated to a S-pseudometric $\mathfrak{m}$ as a family of fuzzy subsets of $X$, $\{g_x\}_{x\in X}$ such that the following conditions hold

1.- $g_x(x) = 1$

2.- $\mathfrak{m}(g_x(y), g_x(z)) \leq \text{Min}\,(\mathfrak{m}(1, g_y(z), (\mathfrak{m}(1, g_z(y)))$ for all $x$, $y$ and $z$ of $X$.

On the other hand, in Höhle [2], "Quotients with respect to similarity relations" the concept of classification w.r.t. a similarity relation in M-SET is introduced. In the second part of this paper, we analyse both concepts and study its relations.

Finally, in the last section, as a consequence of the former analysis we deal with the problem of reducing the number of clusters associated to a T-transitive relation. In this sense, given a fuzzy equality in $X$, we introduce a fuzzy equality in the set $\tilde{P}(X)$ of fuzzy sets of $X$ derived from the one defined in $X$ and we use this relation to stablish an criterion to identify clusters that depends exclusively on the data and selected t-norm. This idea opens a path

[1]Research partially supported by the DGICYT project number PS87-0108

in order to obtain a coherent reduction of the number of clusters (considered as fuzzy sets).

Some of the basic results related with the topics studied in this paper can be found in [4].

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