

SOME RESULTS ON FUZZY BINARY RELATIONS VIA THEIR TRACES

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The main aim of the present paper is to define the right- and left trace of a fuzzy binary relation and to emphasize their role in connection with the basic properties of fuzzy relations. For proofs see [4]. Further related results can be found in [1], [2], [3] and [5].

Let X be a given set, T be a continuous *t-norm* having zero divisors, n be the *strong negation* defined by $n(a) = T^*(a, 0)$ and $S(a, b) = n(T(n(a), n(b)))$ be the *n-dual t-conorm* of T , where $T^*(a, b) = \sup \{ x ; T(a, x) \leq b \}$.

DEFINITION 1. A fuzzy binary relation R on X is called

- reflexive if $R(x, x) = 1$;
- irreflexive if $R(x, x) = 0$;
- symmetric if $R(x, y) = R(y, x)$;
- T -antisymmetric if $x \neq y$ implies $T(R(x, y), R(y, x)) = 0$;
- T -asymmetric if $T(R(x, y), R(y, x)) = 0$;
- S -complete if $x \neq y$ implies $S(R(x, y), R(y, x)) = 1$;
- strongly S -complete if $S(R(x, y), R(y, x)) = 1$;
- T -transitive if $R(x, y) \geq T(R(x, z), R(z, y))$;
- negatively S -transitive if $R(x, y) \leq S(R(x, z), R(z, y))$;
- T - S -semitransitive if
 - $T((R(x, z), R(z, y)) \leq S(R(x, t), R(t, y))$;
 - T -Ferrers relation if
 - $T(R(x, y), R(t, z)) \leq S(R(x, z), R(t, y))$;
 - left-linear if $R(z, x) < R(z, y)$ implies $R(t, x) \leq R(t, y)$;
 - right-linear if $R(x, z) > R(y, z)$ implies $R(x, t) \geq R(y, t)$;
 - linear if it is both left- and right-linear;
 - the left trace of R ($Q^L(R)$) is defined by
$$Q^L(R)(x, y) = \inf_{z \in X} T^*(R(z, x), R(z, y))$$
;
 - the right trace of R ($Q^R(R)$) is defined by
$$Q^R(R)(x, y) = \inf_{t \in X} T^*(R(y, t), R(x, t)). \square$$

- THEOREM 2.** (a) R is reflexive $\Leftrightarrow Q^L(R) \subseteq R \Leftrightarrow Q^R(R) \subseteq R$.
- (b) R is irreflexive $\Leftrightarrow Q^L(R) \subseteq R^d \Leftrightarrow Q^R(R) \subseteq R^d$.
- (c) R is T-transitive $\Leftrightarrow R \subseteq Q^L(R) \Leftrightarrow R \subseteq Q^R(R)$.
- (d) R is negatively S-transitive $\Leftrightarrow R^d \subseteq Q^L(R) \Leftrightarrow R^d \subseteq Q^R(R)$.
- (e) R is T-S-Ferrers relation $\Leftrightarrow Q^L$ is S-complete $\Leftrightarrow Q^R$ is S-complete.
- (f) R is T-asymmetric $\Leftrightarrow R^2 \subseteq (Q^L)^d \Leftrightarrow R^2 \subseteq (Q^R)^d \Leftrightarrow R^2 \subseteq (Q^L \circ Q^R)^d$.
- (g) R is T-S-semitransitive $\Leftrightarrow S(Q^L(x,y), Q^R(y,x)) = 1$.
- (h) R is left-linear $\Leftrightarrow Q^L(R)$ is strongly max-complete.
- (i) R is right-linear $\Leftrightarrow Q^R(R)$ is strongly max-complete.
- (j) R is linear \Leftrightarrow both $Q^L(R)$ and $Q^R(R)$ are strongly max-complete. \square

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