FUZZY PREFERENCES IN LINEAR PROGRAMMING

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This paper presents a practical application of the fuzzy approach to the problems of operations research, namely to problems of linear programming (LP). We deal with the problem of LP with inexact coefficients, which is a practical problem in many applications. Our attention is concentrated on the special area of ranking (comparing) fuzzy values (numbers) in constraints of LP problem. In certain sense this paper continues the paper [5], combining it with the stimulating paper [3].

The problem of LP with inexact coefficients reads as follows:

maximize $\quad c^T x, \; c \in \mathcal{C} \subset E_n,$

subject to

$$A\,x \le b, \quad A \in \mathcal{A} \subset E_{mxn}, \; b \in \mathcal{B} \subset E_m,$$
$$x \ge 0,$$

coefficients A , b , c are taken from the corresponding sets $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$, denoted by capital script letters, x is a crisp decision variable vector. If $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ contain just one element then we have a classical LP problem. *The feasible solution* is defined as a set

$$X = \{x \in E_n^+ \; ; \; A\,x \le \; b\}.$$

If $\mathcal{A}$, $\mathcal{B}$ or $\mathcal{C}$ has more than one element then there arises a problem how to define or understand a corresponding feasible solution set. In operation research we meet four types of "feasible" sets:

$$X_1 = \{x \in E_n^+ \; ; \; \forall\, A \in \mathcal{A}, \; \forall\, b \in \mathcal{B} \; : \; A\,x \le \; b\},$$
$$X_2 = \{x \in E_n^+ \; ; \; \forall\, A \in \mathcal{A}, \; \exists\, b \in \mathcal{B} \; : \; A\,x \le \; b\},$$
$$X_3 = \{x \in E_n^+ \; ; \; \exists\, A \in \mathcal{A}, \; \forall\, b \in \mathcal{B} \; : \; A\,x \le \; b\},$$
$$X_4 = \{x \in E_n^+ \; ; \; \exists\, A \in \mathcal{A}, \; \exists\, b \in \mathcal{B} \; : \; A\,x \le \; b\}.$$

In a *fuzzy linear programming problem* (FLP) we

maximize $\quad \tilde{c}_1 x_1 \tilde{+} \ldots \tilde{+} \tilde{c}_n x_n,$

subject to

$$\tilde{a}_{i1} x_1 \tilde{+} \ldots \tilde{+} \tilde{a}_{in} x_n \;\; \tilde{\varrho}^{con} \; \tilde{b}_i,$$
$$x_j \;\ge\; 0 \;, i=1,..m, j=1,..,n,$$

where $\tilde{c}_j, \tilde{a}_{ij}$, and $\tilde{b}_i$ are fuzzy sets on E_1. Right hand sides of the constraints are compared by a fuzzy preference relation

$\tilde{\varrho}^{con}$ on fuzzy sets. Denoting $\tilde{a}_i \ \tilde{\varrho}^{con} \ \tilde{b}_i \equiv \tilde{\varrho}^{con}(\tilde{a}_i, \tilde{b}_i)$ a *fuzzy feasible solution set* is understood as a fuzzy set on E_n^+. For this purpose we consider 4 constraint fuzzy preference relations:

$$\tilde{\varrho}_1(\tilde{u},\tilde{v}) =$$
$$= \max\{0, \sup\{ \delta\in\langle 0,1\rangle \ ; \ 0\le\lambda\le\delta \Rightarrow \sup[\tilde{u}]_{1-\lambda} \le \inf[\tilde{v}]_{1-\lambda}\}\}$$
$$\tilde{\varrho}_2(\tilde{u},\tilde{v}) =$$
$$= \max\{0, \sup\{ \delta\in\langle 0,1\rangle \ ; \ 0\le\lambda\le\delta \Rightarrow \sup[\tilde{u}]_{1-\lambda} \le \sup[\tilde{v}]_{1-\lambda}\}\}$$
$$\tilde{\varrho}_3(\tilde{u},\tilde{v}) =$$
$$= \max\{0, \sup\{ \delta\in\langle 0,1\rangle \ ; \ 0\le\lambda\le\delta \Rightarrow \inf[\tilde{u}]_{1-\lambda} \le \inf[\tilde{v}]_{1-\lambda}\}\}$$
$$\tilde{\varrho}_4(\tilde{u},\tilde{v}) =$$
$$= \max\{ 0, \sup\{ \delta\in\langle 0,1\rangle \ ; \ 0\le\lambda\le\delta \Rightarrow \inf[\tilde{u}]_{\lambda} \le \sup[\tilde{v}]_{\lambda}\}\}.$$

Here, $[\tilde{w}]_\tau = \{t\in E_1; \ \mu_{\tilde{w}}(t) \ \ge \ \tau \ \}$ .Simpler formulae for symmetric fuzzy numbers are derived. It is shown that when adopting the fuzzy approach to the crisp problem we obtain the same result as when adopting classical approach.

Let $\tilde{u}$, $\tilde{v}$ be fuzzy numbers in a semicommon shape, i.e.

$$\tilde{u}(t) = \tilde{n}\left(\frac{t-u}{\sigma_u}\right), \ \tilde{v}(t) = \tilde{n}\left(\frac{t-v}{\sigma_v}\right),$$

then the above fuzzy preference relations can be expressed in a very simple and explicit formulae as is demonstrated in the paper.

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