Conormes and Fuzzy Measures

in the treatement of quality indices

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Uncertainty measures and the related notions of triangular normes and conormes find very useful applications in modelling the human opinions. As a further particularization they find applications in the manipulation of indices in order to obtain complex indices by simple ones generalizing the usual method of statistic. Several method of aggregating indices are used in statistic, for example in particular if $\mathcal{P}=\{s_1,..,s_n\}$ is a set of indices we can take

$$\max(s_1,..,s_n), \quad \text{and}$$

$$w_1 s_1 + ... + w_n s_n, \quad \text{with } w_i\text{'s weights.}$$

Apart from the weights we must note that if we use indices $s_i \in [0,1]$, then in the two formulas before we use the two conormes maximum and bounded sum. So we can define more in general complex indices as

$$s_1 \perp .. \perp s_n, \quad \text{or with weights} \quad w_1 \mathrm{TS}_1 \perp .. \perp w_n \mathrm{TS}_n.$$

In this way, for two indices, the conormes of Sugeno, or the ones of Yager, or the ones introduced in [Pr], perfectly translate the various way of reasoning as can be shown by

trivial examples.

By the way the use of conormes for the evaluation of complex indices give the translation of an homogeneous behaviour of the objects described by the indices. So using also a result of [C-J] we are compelled to use a "belief function style" in the definition of complex indices.

Connections with many-valued logic are also very useful and fruitful.

References

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