CLASSES OF FUZZY RELATIONS

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Using properties of crisp relations presented by M.Roubens and P.Vincke [3] and also by J.A.Schreider [4] we complete a primary and a secondary classification of fuzzy relations.

As a primary classes we consider families of reflexive, irreflexive, symmetric, asymmetric, antisymmetric, complete, transitive, antitransitive or idempotent fuzzy relations. Together with this properties we introduce also their weaken generalizations (cf.[2]).

As a secondary classes we consider families of tolerance, equivalence, quasi-order, order, linear order or tournament fuzzy relations (cf.[3], [4]). The majority of the above properties of fuzzy relations are equivalent to the respective properties of crisp relations obtained by α-cuts.

The first considered problem is a question does these classes are closed under operations on fuzzy relations such as: conversion, union, intersection, composition, powers and closures:

$$R^{\vee} = \bigvee_{n \geq 1} R^n, \qquad R^{\wedge} = \bigwedge_{n \geq 1} R^n$$

(for a fuzzy relation R on a set $X \neq \emptyset$). A particular answer is contained in Tab.1, where some results are obtained under additional (numbered) conditions:

(1) $R \circ S = S \circ R$,

(2) $R \wedge S^{-1} \leq I$,

(3) $R \wedge S^{-1} = 0_{X \times Y}$,

(4) $R \vee S^{-1} = 1_{X \times Y}$,

(5) $R \circ S \vee S \circ R \leq R \vee S$,

(6) $\operatorname{card} X < \infty$.

Tab.1 ("+" - reads: result belongs to the class of arguments, "(n)" - denotes the same under condition (n)).

CLASS\OPERATION	$R \vee S$	$R \wedge S$	$R \circ S$	R^n	$R^{\vee}$	$R^{\wedge}$
reflexive	+	+	+	+	+	+
irreflexive	+	+				
symmetric	+	+	(1)	+	+	+
antisymmetric	(2)	+				
asymmetric	(3)	+				
complete	+	(4)			+	
transitive	(5)	+	(1)	+	+	(6)
antitransitive	+		(1)	+	+	+
idempotent	(5)	+	(1)	+	+	+

The second problem concerns fuzzy relation equations and inequalities. We ask about sets of all solutions contained in respective classes of relations. First results of this kind were announced in [1]. A more representative list of such results was presented in [2], Chapter 5.

References

[1] J.Drewniak, Fuzzy relation inequalities, in: R.Trappl (ed.), Cybernetics and Systems '88, 677-684, Kluwer Acad. Publ., Dordrecht 1988.

[2] J.Drewniak, Fuzzy relation calculus, Uniwersytet Śląski, Katowice 1989.

[3] M.Roubens, P.Vincke, Preference modelling, Springer Verlag, Berlin 1986.

[4] J.A.Schreider, Equality, resemblance and order, Mir Publishers, Moscow 1975.

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