Logical Basis of Approximate Reasoning with Quantifiers

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The core of vagueness consists in formation of groupings $X = \{x; \varphi(x)\}$ of elements x having a certain property φ. Fuzzy sets are mathematical models of these groupings. The words of natural language can be construed to be the names of the properties φ.

In [1, 2], first-order fuzzy logic (FOFL) is proved to have good formal properties, namely that the completeness theorem holds. Its language may be be extended by additional n-ary connectives o which are interpreted as n-ary operations o : $L^n \longrightarrow L$ provided that the latter keep the so called fitting condition (see [1, 2, 4]) The completeness property of first-order fuzzy logic is not harmed by introducing the additional connectives. This possibility has serious consequences, besides other also on approximate reasoning since it makes possible to put most of its theory on the formal basis of fuzzy logic with sound properties. Moreover, it is possible to introduce easily new connectives from few basic ones. Some important fitting operations on $L = \langle 0, 1 \rangle$ follow:

(a) *product* $a \cdot b$

(b) *bounded sum* $a \oplus b = 1 \wedge (a + b)$

(c) *ϵ-concentration* $\text{CON}_\epsilon(a) = (a - \epsilon)^2$

(d) *ϵ-dilation* $\text{DIL}_\epsilon(a) = 2(a - \epsilon) - (a - \epsilon)^2$

(e) *γ-moderation* $\text{MOD}_\gamma(a) = a + \sin(\gamma - a)$

(f) *γ-increasion* $\text{INC}_\gamma(a) = a + \sin(\gamma)$

We may put $\gamma = 1 - \sin 1$.

These operations can be used as a basis for the new connectives. Let us interpret '•' as '·', '+' as '$\oplus$', Vr as CON_ϵ, Ml as DIL_ϵ, In as INC_γ, and Md as MOD_γ.

As a special case, the linguistic operators introduced in fuzzy set theory can be interpreted as certain unary connectives. For example, very can be interpreted by means of the connective Vr, more or less by Ml, etc. as is usual in fuzzy set theory.

The translation rules in approximate reasoning are the following assignements:

1. A syntagm of the form

   (a) $<adjective> - <noun>$

   (b) $<noun>$ is $<adjective>$

   is assigned an open formula A.

2. Let m be a linguistic modifier interpreted as a unary connective o_m. Then a syntagm $m < adjective > - < noun >$ or $< noun >$ is $m < adjective >$ is assigned the formula $o_m A$.

3. Let A, B be syntagms of the previous form (first case). Then

$$
\begin{array}{lll}
A \text{ and } B & \ldots & A \,\&\, B \\
A \text{ or } B & \ldots & A \vee B \\
IF\ A\ THEN\ B & \ldots & A \Rightarrow B
\end{array}
$$

The linguistic quantifiers (e.g. most, many, etc.) can be interpreted as shorts for certain formulae of FOFL. Let us demonstrate this on the quantifier most. "Most P's are Q's" can be interpreted as "Either all P's are Q's or, at least moderately, there exist P's being Q's but also P's not being Q's". This can be translated as a formula

$$(\mathrm{MS}x)(P(x), Q(x)) := (\forall x)(P(x) \Rightarrow Q(x)) +$$

$$\mathrm{Md}((\exists x)(P(x) \Rightarrow Q(x)) \,\&\, (\exists x)(\neg(P(x) \Rightarrow Q(x))))$$

where MS is a symbol for *most*. On the basis of this definition, we can introduce the sound inference rules of quantity and syllogism. Thus, we may add another translation rule in approximate reasoning, namely:

(f) Let A, B be syntagms of the form (a) or (c), and let q be a linguistic quantifier interpreted as the generalised quantifier Q. Then the syntagm "q A are B" is assigned the formula $Q(A, B)$.

However, let us be aware that the above translation rules cannot be in definite form which works in all cases. The reason consists in the very complicated structure of natural language so that only some expressions of its can be directly translated into formulae.

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