

FUZZY RELATIONS IN ARTIFICIAL INTELLIGENCE

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Fuzzy relations have been studied from several points of view : they can model the idea of a flexible constraint linking variables, the idea of similarity, and the idea of ordering expressing shades of dominance. In this paper we shall review several uses of fuzzy relations within the field of Artificial Intelligence and more specifically in knowledge representation and reasoning.

In knowledge representation, fuzzy relations can express soft notions of granularity, as addressed by Pawlak in the theory of rough sets, for instance. The idea is that objects are described in a granular, finite way by individuals. These granules, linguistically meaningful, refer to partitions of universe of discourses, whose discrimination power may depend upon the context. For instance an individual size will be described on the finite scale [short, medium-sized, tall] in daily life, while a finite set of levels in centimeters may be more appropriate for medical purposes. The problem is then to describe given sets of objects using universes of discourses having a given granularity level. Granularity can be described by a fuzzy similarity relation whose similarity classes form the fuzzy granules representing elementary pieces of knowledge. Formally the problem reads as follows : given a fuzzy set F on a universe U equipped with a similarity relation R , how to build approximations of F defined on the fuzzy quotient set U/R . See [2].

In approximate reasoning, a well-known problem is the following : given a set of fuzzy rules $\{ \text{if } x \in A_i \text{ then } y \in B_i \}_{i=1,n}$ describing constraints linking two variables x and y on referentials X and Y , how to build the fuzzy relation R representing this set of rules, and how to compute the value of y , given that x is restricted by a fuzzy set A . This is the extension of deductive reasoning to fuzzy knowledge. Usually, relation R can be obtained by solving a set of fuzzy relational equations involving $\{A_i, B_i, i = 1,n\}$. However there are several possible ways of stating this problem that correspond to various meanings of the rules [1]. Moreover given that x is restricted by A , the set of deduced values of y may also be defined in several ways : one may characterize either the set of y 's related to at least one value of x in A , or the set of y 's related to *all* values in A , through the fuzzy relation implicitly at work in the rules. These notions of inference can be generalized using quantifiers, to solve problems such as the following one : given a fuzzy relation R , and a fuzzy set A of values of x , what is the fuzzy set of values of y related to a given (fuzzy) proportion of values in A [3].

Fuzzy relations have also been used in the past for abductive reasoning : R represents a causal relation relating causes to observations. The abductive reasoning problem is then : given a fuzzy set of observations A , find the fuzzy set(s) of causes B which explain the observations A . Clearly this is a matter of solving a fuzzy relational equation. But in the past literature, due to the presence of possibly several solutions to the equation $B \circ R = A$, the application of fuzzy relations to diagnosis has often been confusing so that it is difficult to interpret the results of the model. The meaning of various possible kinds of solutions to the abduction problem is discussed in [3].

Lastly fuzzy relations have been recently used by the authors for the modelling of order-of-magnitude relations such as "in the neighborhood of" and "negligible in front of", that are very common in qualitative reasoning. The aim of this work is to build a formal inference system handling such concepts, whose semantics is approximate computation with real numbers. The emphasis is a rigorous but flexible interface between the numerical level and the symbolic level that must remain consistent with one another. The involved fuzzy relations are similarity and fuzzy ordering relations that are respectively "weakly" and "strongly" transitive, and inferences at the symbolic level correspond to various compositions of these fuzzy relations. See [4] for details.

From a mathematical point of view these applications are based on the following notions and results that are described in greater details in the papers of the reference list :

- the existence of dual \sup - $*$ and \inf - $*$ \rightarrow composition of fuzzy relations, where $*$ is a conjunctive operation, and $*$ \rightarrow is the implication operation obtained from $*$ by residuation [3],
- the existence of two adjoint families of conjunctive operations : the triangular norms, and a non-commutative family of conjunctions which play symmetric roles with respect to the two definitions of multiple-valued implications, i.e. via residuation or by means of the clausal form $(a \rightarrow b = \neg a \vee b)$ [5],
- the resolution of fuzzy relational equations. While this is well-known in the case of deductive and abductive reasoning methods, the use of fuzzy relation equations is less known for the problem of finding similarity classes of a max-triangular norm transitive similarity relation, independently solved by Valverde in the early eighties. We show that this problem can also be solved by a straightforward application of standard results in fuzzy relational equations, applied to the transitivity property, i.e. solving for X the equation $R \circ X \subseteq R$, and acknowledging R as a solution of this equation [2],
- the possibility of computing the composition of fuzzy relations on continuous universes by means of fuzzy interval calculations [4].

The results mentioned in this abstract correspond to the following papers

- [1] Dubois D., Prade H. A typology of fuzzy "if... then... rules. Proc. of the Inter. Fuzzy Systems Association (IFSA) Congress, Seattle, Wash., August 6-11, 1989, 782-785.
- [2] Dubois D., Prade H. Fuzzy rough sets and rough fuzzy sets. Int. J. of General Systems 17(2-3), 1990, 191-209.
- [3] Dubois D., Prade H. Upper and lower images of a fuzzy set induced by a fuzzy relation : applications to fuzzy inference and diagnosis. In : Tech. Repo. LSI n°265 (Univ. P. Sabatier, Toulouse), 1987. Revised version to be published in Information Sciences.
- [4] Dubois D., Prade H. Order-of-magnitude reasoning with fuzzy relations. Revue d' Intelligence Artificielle, 4(4), 1989, 69-94.
- [5] Dubois D., Prade H. A theorem on implication functions defined from triangular norms. Stochastica, VIII(3), 1984, 267-279 and in : BUSEFAL n° 18, I.R.I.T., Univ. P. Sabatier, Toulouse, 1984, 33-41.

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