INFERENCE THROUGH RELATIONS IN MULTIPLE-VALUED AND FUZZY LOGICS*

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ABSTRACT.

The notion of "institution" (Goguen & Burstall 83), using the language of Category theory, formalizes the intuitive notion of a logical system from a model-theoretic point of view, that is, taking the satisfaction relation between models and sentences as basic.

Informally, an institution consists of:

- a collection of signatures (which are vocabularies for use in constructing sentences in a logical system) and signature morphisms, together with each signature S,
- a set of S-sentences,
- a set of S-models,
- an S-satisfaction relation of S-sentences by S–models, such that when you change signatures, with a signature morphism, the satisfaction relation between sentences and models changes consistently.

Many of classical logical Systems (First order logic, Equational logic, Horn Clause logic, ...) are examples of institutions.

On the other hand, multiple-valued logics are built on a set of predicate and connective symbols and on a set of truth-values (Rasiowa; 1974. Rescher; 1969). Predicate and connective symbols are used to build sentences and truth-values are used to evaluate them. Usually, the truth-values set is structured as an algebra similar to the algebra of sentences with two distinguished elements ('True', 'False'). Valuations are morphisms from the free algebra of sentences to the chosen truth-values algebra.

In a recent work (Agustí-Cullell et al., 1990), multiple-valued logics have been structured as families of institutions, each one being indexed by a class a truth-values algebras. There, the interpretation of predicate symbols are fuzzy relations and, in order to have a classical satisfaction relation, truth-values have been considered as part of the sentences, i.e., sentences are ordered pairs (s,W) where s is a sentence in the classical sense, and W is a subset of truth-values. On the other hand, it has been proved that each morphism of truth-values algebras defines a corresponding institution morphism .

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Following this line, in this communication, we study the formalitzation of the fuzzy logic in the institutional framework, and some inference schemes which can be obtained from the semantical deduction defined in the Institution. Here we understand "fuzzy logic" as a logic whose truth-values (henceforth referred as "fuzzy truth-values") are fuzzy subsets of elements of an above mentioned truth-values algebra. The algebra operations are extended to fuzzy truth-values by means of the so-called extension principle (Dubois, Prade; 1988).

In formalizing fuzzy logic as institution, the main differential characteristics with respect to multiple-valued logic are:

- sentences are ordered pairs (s,τ), where τ is a fuzzy truth-value,
- models incorporate an indexed family of fuzzy subsets giving the membership degree of individuals to model subdomains,
- the satisfaction relation between models and sentences is defined comparing the fuzzy truth-values τ with a generalized compatibility function between two fuzzy sets, one representing the interpretation of the formula s, and the other one, obtained with the family of fuzzy subsets representing the individual domain of the model.
- morphisms of truth-values algebras do not directly extend to fuzzy logic institutions morphisms, as it was the case of MVL-institutions.

KEYWORDS: Multiple-valued Logics, Institution, Truth-Values Algebra, Fuzzy Logic

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