REPRESENTING MV-ALGEBRAS BY FUZZY SETS

Antonio Di Nola
Math. Inst. of Architecture, Univ. of Naples
via Monteoliveto 3, 80134 Naples, Italy.

Introduction

An MV-algebra is an algebra $A = (A,+,\bullet,*,0,1)$ where $(A,+,0)$ is an abelian monoid, and the following identities hold: $x+1=1$, $x^{**}=x$, $0^*=1$, $x\bullet y=(x^*+y)^*$, $(x^*+y)^*+y=(y^*+x)^*+x$. Replacing y by 1 in the last identity, one easily obtains $1=x^*+x$.

Chang [2,3] invented the MV-algebras in order to give an algebraic proof of the completeness theorem of the infinite-valued logic of Lukasiewicz. Recently MV-algebras received an increasing attention after the introduction of the functor Γ by D. Mundici [6]. As proved in [7,3.9], Γ is a categorical equivalence between abelian l-groups with strong unit, and MV-algebras.

In this paper, MV-algebras are studied "per se". We state a representation theorem for MV-algebras which makes use of a kind of MV-algebras constructed over non-standard models of [0,1].

Representation of MV-algebras by functions

Following [2], we stress that the unit interval [0,1] becomes an MV-algebra, defining $x+y=\min(1, x+y)$, $x^*=1-x$, and

$x \bullet y = \max(0, x+y-1)$, for all $x,y \in [0,1]$. Further, any ultrapower (see [4]) of $[0,1]$ is an MV-algebra denoted by $[0,1]^*$.

Theorem 1

For every MV-algebra A there exists an ultrapower $[0,1]^*$ such that A can be embedded into $\pi_J\{[0,1]^* \;/\; J \in \mathrm{Spec}A\}$.

Following L.P.Belluce [1, Theorem 4, p.1359] we can paraphrase the above theorem by saying that each MV-algebra A can be represented as an MV-algebra of non-standard fuzzy sets (see [5]) from SpecA to $[0,1]^*{}_A$.

References

[1] **L.P.BELLUCE, Semisimple algebras of infinite-valued logic, Canadian J.Math. 38(1986)1356-1379.**

[2] **C.C.CHANG, Algebraic analysis of many valued logics, Trans.Amer.Math.Soc.88(1958)467-490.**

[3] **C.C.CHANG, A new proof of the completeness of the Lukasiewicz axioms, Trans.Amer.Math.Soc.93(1959)74-80.**

[4] **C.C.CHANG, H.J.KEISLER, Model Theory, North-Holland,1973.**

[5] **A.DI NOLA, G.GERLA, Non-standard fuzzy sets, Fuzzy Sets and Systems 2(1986)173-181.**

[6] **D.MUNDICI, Mapping abelian l-groups with strong unit one-one into MV-algebras, J.Algebra 98(1986)76-81.**

[7] **D.MUNDICI, Interpretation of AF C*-algebras in Lukasiewicz sentential calculus, J.Functional Analysis 65(1986)15-63.**