CHAOTIC FUZZY SYSTEMS

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Abstract: The concept of chaotic fuzzy system is introduced and briefly analysed. Computer simulations of chaotic fuzzy systems are presented.

Keywords: fuzzy system; fuzzy feedback; fuzzy oscillators; chaos.

1. Introduction

It is well known that a chaotic (crisp) system is defined as a system that is assimptotically unstable and which unstability is neither periodical or cvasi-periodical /1/.

It is natural to ask if there exists 'fuzzy chaos'. Once this concept rigorously defined -- at least: as rigorously as in the crisp case, because even in this case the definition is notorously imprecise -- one can ask how a chaotic fuzzy system can be build.

The present paper addresses mainly these two questions.

2. The 'fuzzy chaos'

To define the fuzzy chaos we shall rely on two concepts previously introduced /2/, /3/, namely on the definitions of fuzzy signals and of state-transitions graphs of fuzzy systems. Briefly, a fuzzy system is chaotic iff its output neither reaches a stable (or assimptotic stable) value, nor is it periodical or cvasi-periodical, in the sense of fuzzy signals, i.e. iff its time-dependent membership function is neither stable, nor periodically changing.

In the case it is easy to evidence the state of the fuzzy system -- as for discrete-time systems-- it could be useful to define the chaotic character with respect to the state-transitions graph. (Remember that the states are fuzzy, usually expressed by linguistic degrees). A discrete-time fuzzy system is chaotic iff its graph is neither cyclic, nor finite. The extension to continuous-time fuzzy systems leads to a similar deffinition for the trajectory of the state.

3. Fuzzy feedback systems

A fuzzy system provided with a loop bringing (a function of) the output back to the input was briefly named a 'feedback fuzzy system'. If the feedback pass includes a fuzzy system, then the overall structure will be breifly named a fuzzy feedback system.(See for the background /1/, /4/). In this paper, the fuzzy system in the feedback loop will be considered to be of the configurations sketched in Figure 1 a, b.

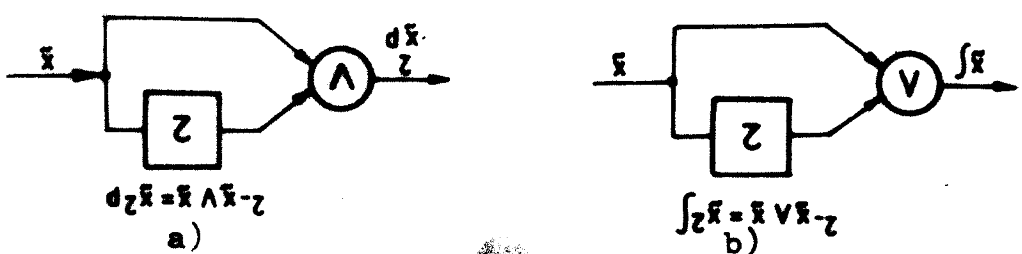

Fig. 1: The feedback pass configurations

Incidentally, note that the system in Fig. 1 a) acts as some kind of 'fuzzy derivator', while the system in Fig. 1 b) acts as a fuzzy integrator.

Using the above systems in the feedback loop of a fuzzy system R, one gets the fuzzy feedback system sketched in Figure 2 a) and respectivelly b).

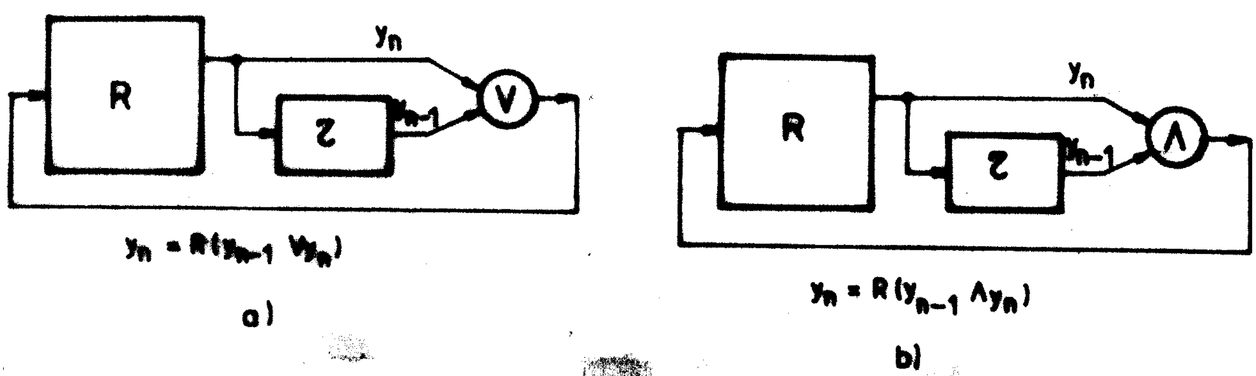

Fig. 2: Feedback systems configurations

In what follows, only the system sketched in Fig. 2 a) will be discussed, although a similar behaviour can be obtained using the system in Fig. 2b).

4. Computer simulation of the chaotic fuzzy system

To generate an entire class of fuzzy feedback systems in simulation, an alteration of the system presented in Fig. 2 a) was used, namely the output of the join node was first defuzzified and then multiplied by a constant . This constant, as well as the original conditions (see /1/, /4/) and the membership functions defining the inputs and the outputs can be changed in the program. The membership functions were choosed to be of triangular or trapezoidal shape. The program for simulation was written in PASCAL.

Computer simulations evidenced all three types of behaviours for the systems in the tested class: asimptotically stable, oscillatory and chaotic behaviours, depending on the parameters of the system. Note that even if the rules describing the system R are fixed, (only 'linear' systems /1/,/4/ were used), all three regimes still occur.

The existence of a chaotic regime is insured by the fact that the system R in the main signal path has a cyclic transitions graph and by the property of the feedback path, consisting in the possibility to continuously change the fuzzy signal presented to its input. (Above, by the existence of a cyclic grapgh of the system R, we are understanding, more precisely, the fact that the system R with a pure delay feedback presents such a state-transitions graph /1/,/4/).

It is worth to mention that a large class of fuzzy feedback systems present chaotic behaviour:

<u>Proposition</u>. Let be a fuzzy feedback system consisting of: i) a 'linear' system with triangular-valued inputs and outputs, such as a) at least five linguistic degrees exist (for the input and the output; the same membership functions are assumed for the input and the output); b) the triangles representing the membership functions overlap such as, for any three successive triangles T_k, T_{k+1}, T_{k+2}, T_k and T_{k+2} overlap exactly in one point; c) the system with a purely delay-type feedback presents a cyclic graph of states transitions;

ii) a feedback loop consisting in a fuzzy integrator (Fig. 1 b) followed by a defuzzifier and crisp system with linear characteristic function (i.e. which output is a linear function w = az + b of its input z). Then:

There exists a couple of constants (a,b) such that the fuzzy feedback system is chaotic.

The proof will be given elsewhere.

An example of chaotic behaviour is presented in Fig. 3. A seven linguistic degree system R was used, with triangular membership functions centered in -3,-2,-1,0,1,2,3.

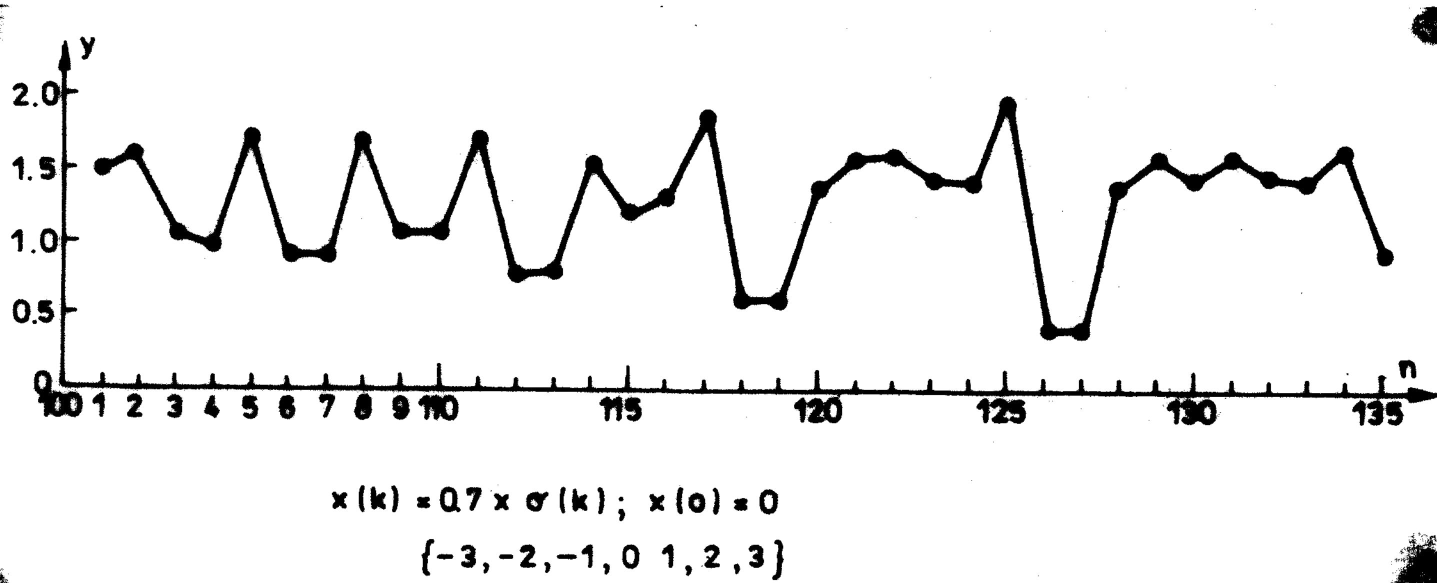

Fig. 3: Chaotic behaviour

Concluding, a quite large class of fuzzy systems were proved to exhibit a chaotic behaviour. Such systems could be useful in generating random signals,and could find applications in communications engineering and computer technology.

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