Some New Methods of Group Sequencing And Its Structure Reforming

YuBin Zhong

(Maths Dept. Guang Zhou Teachers College)

ABCTRACT. This paper is proceeded from conventional sequencing we have introduced some new methods of sequencing and illustrated their efficiency by examples, then we have given two comparable factors and have analysed that it is possible to reform the group structure in terms of these two comparable factors. This paper has some pratical value.

§1 Conventional Sequencing Method

At present, following method is adopted in most sequencing:

I The case of single index:

Let universe $U = \{u_1, \cdots, u_n\}$, $u_i = (u_{i1}, \cdots, u_{ij_i})$ $(1 \le i \le n)$ their scores (quantity) from single index c are $a_{i1}, \cdots, a_{ij_i}$ put $a_i = \sum_{k=1}^{j_i} a_{ik}$ $\cdots$ (1) or $a_i = (\sum_{k=1}^{j_i} a_{ik})/j_i$ $\cdots$ (2). Here $i = 1, 2, \cdots, n$. Then we can obtain the sequencing of $u_1, \cdots, u_n$ only by comparing the magnitude of $a_1, \cdots, a_n$.

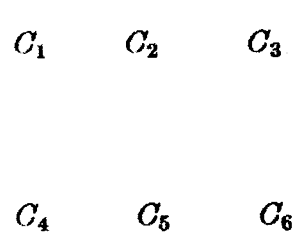
II The case of several indexes.

Consider several indexes $c_1, \cdots, c_n$ and determination weight $b_1, \cdots, b_m$, By (1) we can calculate the u_i quantity $a'_{ij} = \sum_{k=1}^{j_i} a_{ik}$ corresponding to c_j. Put $a'_i = \sum_{j=1}^{m} b_j a'_{ij}$ $\cdots$ (3) or $a'_i = (\sum_{j=1}^{m} b_j a'_{ij})/m$ $\cdots$ (4). Only if we could determine the magnitude of $a'_1, \cdots, a'_n$, we could determine the sequencing of $u_1, \cdots, u_n$.

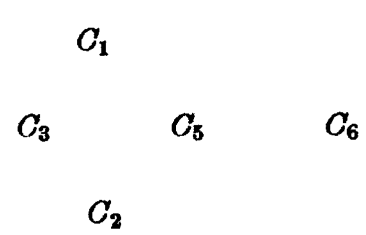
We call these two sequencing methods to be conventional sequencing methods. But in most cases, c_i can also be decomposed into several indexes $(c_{i1}, \cdots, c_{ik_j})$. Indexes $c_1, \cdots, c_m$ may also be alternative, so the partition of index itself has fuzziness and variety. Ordinarily, the decomposition of index c_i may be unreasonable the weight isn't determined easily. Therefor the credibility is reduced actually. However the man brain expecially expert's has unique synthesis ability in the partition of fuzy index that is complicated in somewise and weight determination. Next we considerr from the characteristic of Fuzzy Cognitive Maps.

§2 Determine index and weight by Fuzzy Cognitive Maps.

Fuzzy Cognitive Maps is often abbreviated to FCM. A FCM is a Fuzzy map with sign and fed-backing. The node denotes the Fuzzy Set. Fuzzy directal edges denote partial causal relative. "+" refers to causal increase and "–" to causal decrease. We survey n authorities, they are asked to use a FCM to denote their suggestion on c_i decomposing or weight. According to priciple of knowledge composing in [2], we compose a new FCM. This FCM synthesis all experts' experimence and more reasonability. For example, synthetical map is Figure 1. Index $c_1, \cdots, c_6$. Its conjuction degree is side weight. If we think that side weight less than or equal to $\frac{1}{4}$ is too weak and it can be neglected. We can obtain $\frac{1}{4}$ strict cut-map of Figure 1 (Figure 2). Then on this standard. Synthesis expert FCM is much simpler than before.



(Figure 1)



(Figure 2)

The index can be simplifided to c_1, c_2, c_3, c_5, c_6. By the priciple of knowledge composing, we can determine the weight also. In refer to sequencing problem on given universe. After determining index and weight, we can sequence by different methods. Next we introduce some new sequencing methods.

§3. Determine group sequencing by linear order.

After better solving index devision and weight determination. We can discuss the concrete method in sequencing.

Assume m indexes, in every index we determine sequencing L_i $(i = 1, \cdots, m)$ of universe $\{u_1, \cdots, u_n\}$. By [4], L_i forms Linear order. In order to introduce two new sequencing methods, we introduce following conception.

DEFINITION 1. *Let $U = \{u_1, \cdots, u_n\}$ be a universe. L_i is linear order (evaluation) on U. For any $u \in U, c_i(u)$ denotes sequence number of u in L_i.*

Put $c(u) = \sum_{i=1}^{m} c_i(u)/m$. $c(u)$ is called position number of U.

DEFINITION 2. *Let L_1, L_2 be two linear orders on U. The distance of L_1 and L_2 is defined by $d(L_1, L_2) = \sum_{i=1}^{m} |c_1(u_i) - c_2(u_i)|$, here $|c_1(u_i) - c_2(u_i)|$ denotes the difference of sequence number of element u_i on L_1 and L_2. Therefor definition of $d(L_1, L_2)$ is very audio-visual.*

DEFINITION 3. *Let $L_1, \cdots, L_m$ be m linear orders on U. L is any linear order on U. L^* is a linear order.If $\sum_{i=1}^{m} d(L^*, L_i) = \min_{L} \sum_{i=1}^{m} d(L, L_i)$, the L^* is call the best evaluation of $L_1, \cdots, L_m$. This method is similar to the method of minimum distance that we are acquainted. So we call this method to be the method of minimun distance of sequencing.*

The similarity of two linear orders can be denoted by correlation coefficient exception by distance. From Multivariate Statistical Analysis we know a variety of correlation coefficient that can be used. If we use the cosine of angle, we can obtain following definition.

DEFINITION 4. *Let universe $U = \{u_1, \cdots, u_n\}$. L_1 and L_2 are two linear orders on U. Their sequencing number are $c_1(u)$ and $c_2(u)$. correspondingly, then the correlation* *coefficient of L_1 and L_2 is defined by* $\rho(L_1, L_2) = \dfrac{\sum_{i=1}^{n} c_1(u_i)c_2(u_i)}{\sqrt{\sum_{i=1}^{n} {c_1}^2(u_i) \sum_{i=1}^{n} {c_2}^2(u_i)}}$

DEFINITION 5. *Let $L_1, \cdots, L_m$ be m linear orders on $\mathcal{U}$. L is any linear order on $\mathcal{U}$. L^* is a linear order. If $\frac{1}{m}\sum_{i=1}^{m} \rho(L^*, L_i) = \max_L(\frac{1}{m}\sum_{i=1}^{m} \rho(L, L_i))$, then we call L^* to be the best evaluation of $L_1, \cdots, L_m$. Corresponding method is call the method of maximum relation.*

Now let us use an example to illustrate the efficiency of these two methods. We take the example in [6]. Index is given. Three agriculture product plan and every economical index is as following:

economical index	plan 1	plan 2	plan 3
kg/mu	1850	1400	2150
yuan/kg	4.8	4.1	6.5
manpower/mu	35	22	52
yuan/mu	125	115	90
fertility of land	4	4	2

In order to make the method convenient. We correct the fertility of land in plan 2 to be grade 3 from grade 4. From the calculation in [6] we know this turn will not change the sequencing of u_1, u_2, u_3. According to single economical index we obtain result as following:

$$L_1(Kg/mu) : u_3\, u_1\, u_2 \qquad L_2(yuan/Kg) : \; u_2\, u_1\, u_3$$

$$L_3(manpower/mu) : u_2\, u_1\, u_3 \qquad L_4(yuan/mu) : u_1\, u_2\, u_3.$$

$$L_5(fertility) : u_1\, u_2\, u_3$$

1. The method of minimum distance:

Let universe $\mathcal{U} = \{u_1, u_2, u_3\}$, its all possible linear orders are:

$$L_1' : u_1u_2u_3, \quad L_2' : u_1u_3u_2 \quad L_3' : u_2u_1u_3$$
$$L_4' : u_2u_3u_1, \quad L_5' : u_3u_1u_2 \quad L_6' : u_3u_2u_1$$

(1)Calculate the distance sum of L_1' and $L_1 \sim L_5$:

We can easily obtain:

$$d(L_1', L_1) = 4, \quad d(L_1, L_i) = 2 \ (i = 2, 3), \quad d(L_1', L_i) = 0 \ (i = 4, 5)$$
$$\therefore \sum_{i=1}^{5} d(L_1', L_i) = 4 + 2 + 2 + 0 + 0 = 8$$

(2) Calculate the distance sum of L_i' $(i = 2, 3, \cdots, 6)$ and $L_1 \sim L_5$:

$$\sum_{i=1}^{5} d(L_2', L_i) = 14 \quad \sum_{i=1}^{5} d(L_3', L_i) = 8 \quad \sum_{i=1}^{5} d(L_4', L_i) = 16$$
$$\sum_{i=1}^{5} d(L_5', L_i) = 16 \quad \sum_{i=1}^{5} d(L_6', L_i) = 18 \ .$$
$$\because \quad \sum_{i=1}^{5} d(L^*, L_i) = \min_j \sum_{i=1}^{5} d(L_j', L_i)$$

$\therefore \quad L^* = L_1'$ or L_3'

Therefor the central opinion obtained by the method of minimun distance is linear order : $L_1' = u_1u_2u_3$ or $L_3' = u_2u_1u_3$.

2. The method of Maximun relation.

$$\sum_{i=1}^{3} c_j{}^2(u_i) = 1^2 + 2^2 + 3^2 = 14 \qquad (j = 1, 2, 3, 4, 5)$$

$$\sum_{i=1}^{3} \left(c_j'(u_i)\right)^2 = 1^2 + 2^2 + 3^2 = 14 \qquad (j = 1, 2, 3, 4, 5, 6)$$

(1) Calculate the correlation coefficient of L_1' and $L_1 \sim L_5$.

$$\rho(L_1', L_1) = \frac{\sum_{i=1}^{3} c_1'(u_i)c_1(u_i)}{\sqrt{\sum_{i=1}^{3}\left(c_1'(u_i)\right)^2 \sum_{i=1}^{3} c_1{}^2(u_i)}} = \frac{11}{14}.$$

Similarily

$$\rho(L'_1, L_2) = \frac{13}{14} \qquad \rho(L'_1, L_3) = \frac{13}{14} \qquad \rho(L'_1, L_4) = \frac{13}{14} \qquad \rho(L'_1, L_5) = \frac{12}{14}$$

$$\therefore \quad \tfrac{1}{5}\sum_{i=1}^{5} \rho(L'_1, L_i) = \frac{61}{70}$$

(2) Calculate the correlation coefficient of Li and $L_1 \sim L_5$:

$$\frac{1}{5}\sum_{i=1}^{5} \rho(L'_2, L_i) = \frac{61}{70}, \qquad \frac{1}{5}\sum_{i=1}^{5} \rho(L'_3, L_i) = \frac{64}{70}, \qquad \frac{1}{5}\sum_{i=1}^{5} \rho(L'_4, L_i) = \frac{59}{70}$$

$$\frac{1}{5}\sum_{i=1}^{5} \rho(L'_5, L_i) = \frac{56}{70}, \qquad \frac{1}{5}\sum_{i=1}^{5} \rho(L'_6, L_i) = \frac{55}{70}$$

$$\because \quad \frac{1}{5}\sum_{i=1}^{5} \rho(L^*, L_i) = \max_{L'} \cdot \frac{1}{5}\sum_{i=1}^{5} \rho(L', L_i) = \frac{64}{70}$$

$$\therefore \quad L^* = L'_3 = u_2 u_1 u_3.$$

The result by these two methods is somewhat different with that by [6], but by carefully analysis, this result is reasonable.

§4. Method of nearness in group sequencing

Here, we introduce an efficient new method in group sequencing method of nearness.

1. single index c_1:

DEFINITION 6. *Let $u_i = (u_{i1}, u_{i2}, \cdots, u_{ij_i}$, every component is a quantity in [0,1]. Then u_i is a Fuzzy subset. If $(u_{ij_1} > u_{ij_2}) \iff (a_{ij_1} > a_{ij_2})$, then u_i is called increasing index. Correspondingly the best subset u_i^* is $(1, \ldots, 1)$. If $(u_{ij_1} > u_{ij_2}) \Leftrightarrow (a_{ij_1} < a_{ij_2})$, then u_i is called decreacing index. The best subset u_i^* is $(0, \ldots, 0)$.*

DEFINITION 7. *If* $\quad N(u_i, u_i^*) > N(u_j, u_j^*) \iff u_i > u_j$

$$N(u_i, u_i^*) = N(u_j, u_j^*) \iff u_i = u_j$$

there, $N(u_i, u_i^*)$ denotes nearness of u_i and u_i^* .

Obviously, this definition is naturally and has strong visual property.

Then, it is naturally to put forward the selection problem of nearness N. Concrete analysis should be made in concrete conditions. For example, if group sequencing is based on the representation of magnitude of man's ability. We should select the nearness that can response small difference in high ability and large difference in low ability. Paper [7] pointed out that Euclid nearness has this character of small action in high part and large action in low part. So we can select the Euclid nearness: $N(u_1, u_2) = 1 - \frac{1}{\sqrt{n}}[\sum_{i=1}^{n}(u_{1i} - u_{2i})^2]^{\frac{1}{2}}$.

Give an example : Assume two students take three examinations. Their scores is 100, 40, 70 and 75, 70, 55. Therefor, we can structure the fuzzy subsets $u_1 = (1, 0.40, 0.70)$, $u_2 = (0.75, 0.7, 0.55)$, $u_i^* = (1,1,1) \quad (i = 1,2)$. so

$$N(u_1, u_1^*) = 1 - \frac{1}{\sqrt{3}}(0^2 + 0.6^2 + 0.3^2)^{\frac{1}{2}} = 1 - \sqrt{0.45}/\sqrt{3}$$

$$N(u_2, u_2^*) = 1 - \frac{1}{\sqrt{3}}(0.25^2 + 0.3^2 + 0.45^2)^{\frac{1}{2}} = 1 - \sqrt{0.355}/\sqrt{3}$$

$$N(u_1, u_2^*) < N(u_2, u_2^*)$$

$$so \quad u_1 < u_2.$$

Although this result is different with conventional method. This method is useful for reference and reasonable. Now we apply this method on several indexes.

2. Several indexes : $c_1, \cdots, c_m$

DEFINITION 8. *General nearness $N(N_1, N_2, \cdots, N_p)$ is a function of p-variable, satisfies*

(1) $N(x, x, \ldots, x) = x$, *for all* $x \in [0,1]$

(2) *If* $N_1 \geq N_1', N_2 \geq N_2', \cdots, N_p \geq N_p'$ *(N_i, N_i' are both nearness)* *and at least a strictly inequality holds, then* $N(N_1, \cdots, N_p) > N(N_1', \cdots, N_p')$.

COROLLARY 1. $\min_{1 \leq i \leq p}\{N_i\} \leq N(N_1, \cdots, N_p) \leq \max_{1 \leq i \leq p}\{N_i\}$

COROLLARY 2. $0 \leq N(N_1, \cdots, N_p) \leq 1$.

Given the relatively reasonable indexes $c_1, \cdots, c_m$ and weights $b_1, \cdots, b_m$ correspondingly ($b_i \geq 0, and \sum_{i=1}^{m} b_i = 1$) that are determined by FCM. We assume the nearness between u_i and u_i^* referring to c_j is N_{ij}. Put $N(N_{i1}, \cdots, N_{im}) = \sum_{j=1}^{m} b_j N_{ij}$, then there is a concrete general nearness. We can give group sequencing according to the magnitude of $N(N_{i1}, \cdots, N_{im})$. Example won't be give here.

§5. Reforming of the group structure

We have introduced some methods of group sequencing before. But people discuss group sequencing not only in order to sequence, but also try to give more convinced suggestion. Then we can reform the group structure and make the group have more perfect structure. Now we use methods of nearness and statistic in our discussion.

I. Urgent coefficient of system reforming:

Put $M_i = 1 - N(u_i, u_i^*)$, then:

The smaller N is, the larger M_i is. The output function is weaker, and it is more urgent to reform the system u_i.

The larger N is, the smaller M_i is. The output function is stronger, and it is less urgent to reform the system u_i.

Therefor we call M_i to be urgent coefficient of system reforming.

II. To fuzzy subset $u_i = (u_{i1}, \cdots, u_{ij_i})$, we consider variance $d_i = \frac{1}{j_i - 1} \sum_{k=1}^{j_i} (u_{ik} - \overline{u_i})^2 \in [0,1]$, here $\overline{u_i} = \sum_{k=1}^{j_i} u_{ik}/j_i$. $j \geq 2$, Now we come to prove that: $d_i \in [0, 1]$.

LEMMA 1. *If $d_i > 1$, then there is at most one $k(1 \leq k \leq j_i)$, so that $(u_{ik} - \overline{u_i})^2 \leq 0.5$.*

LEMMA 2. *If $|u_{ik} - \overline{u_i}| > \sqrt{0.5}$. for all k $(1 \leq k \leq j_i - 1)$, then*
either $(u_{ik} - \overline{u_i}) > \sqrt{0.5}$. for all k $(1 \leq\leq j_i - 1)$
or $(u_{ik} - \overline{u_i}) < -\sqrt{0.5}$. for all k $(1 \leq k \leq j_i - 1)$

These two Lemma can rather easily be proved. With them we can deduce the following theorem by contradiction. Because of the length, we omit the proofs.

THEOREM. *If $u_{ik} \in [0,1], 1 \le k \le j_i$, $j_i \in N$. and $j_i \ge 2$. then*

$$d_i = \frac{1}{j_i - 1} \sum_{k=1}^{j_i} (u_{ik} - \overline{u_i})^2 \in [0,1]$$

Here, d_i denotes the degree of every component u_i departuring the average value, ie. individual difference degree of function in u_i. If d_i is larger, the individual function difference of u_i is larger. It is more urgent to reform partial individuality. Contrarily, the smaller d_i is, the smaller the individual function difference is. It is less urgent to reform the individuality with rather weak function. We call d_i to be urgent coefficient of partial reforming.

Having these two comparable factors, we include the following regularities:

1) M_i and d_i are high, ie., the whole function output of u_i is lower, the difference of function output between individuality is large, then except that we should manage to enhance the individual quality which power is weak in u_i, we should reform the whole working system of u_i.

2) Value M_i in u_i is partially high, value d_i is partially low. It denotes the whole function output is lower, but individual difference is smaller, therefor we should reform the system on u_i. The problem will not be solved only by the method of promoting the backward, collective professional education and training on the group must be carried out.

3) Value M_i is suitable, d_i is partially high. It denotes that the whole function output is high, but the individual difference is large. Then, the primary work is to enhance the weaker individual quality by promoting the backward. So as to diminish difference.

4) Both M_i and d_i are not partially high. It denotes the whole function output is stronger. Still higher criterion can be made according to actuality so as to enhance individual quality and group output.

Interested reader can discuss further according to the comparable factors M_i and d_i.

Refference

[1]. Wang Peizhuang: Theory of Fuzzy Set and its Application, Shanghai Science and Technology Press, 1983

[2]. Yubin Zhong, Zhen Jin : Fuzzy Cognitive Maps and Its Application. The Paper Set of the 5'Annals meeting of Fuzzy Maths and System , Chinese System Engineering Society, 1990.

[3]. Luo Chengzhong: Introductory of Fuzzy Sets, Beijing Normal University Press, 1989.

[4]. Wu Wangming: Method of Applied Fuzzy Sets, Beijing Normal University Press, 1985.

[5]. Zhang Yaoting: Introductory of Multivariate Statistical Analysis, Science and Technology Press, 1982

[6]. He Zhongxiong: Fuzzy Maths and Its Application, Tianjin Science and Technology Press, p. 234(1983).

[7]. Potencial Science, 1987(2).