A Note on the Limit of Sequenees of Grey Numbers

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Abstract

In this note, the following results are proved:

1. If $\lim_{n\to\infty} x_n = a$, $\lim_{n\to\infty} y_n = b$, then $\lim_{n\to\infty} x_n y_n = ab$.

2. If $\lim_{n\to\infty} x_n = a$, $\lim_{n\to\infty} y_n = b$ and $O \bar{\in} [P[b], Q[b]]$, then $\lim_{n\to\infty} \frac{x_n}{y_n} = \frac{a}{b}$.

Key words

Classical rational grey number, Sequence of grey numbers, Metric, Set of grey points, Limit.

In Wu Heqin et al . [1], the concepts of classical retional grey numlers is introduced. A classical grey number is either a classical grey number of the interval type or a classical grey number of the information type. The aim of this note is to discuss the operational reles of limits of sequences of grey numbers.

Let $R = (-\infty, +\infty)$ be the set of all real numbers.

Let $\underset{\sim}{R}$ be the set of classical retional grey numbers,and $\underset{\sim}{S}$ be the set of grey points. i.e.

$\underset{\sim}{R} = \{[a, b], \widetilde{[a, b]} | a \leqslant b, a, b \in R\}$

$\underset{\sim}{S} = \{(a, b, c) | a, b, c \in R, a \leqslant b, c \in \{0, 1\}\}$

It is known that there exists a one—to —one function Φ from $\underset{\sim}{R}$ onto $\underset{\sim}{S}$: $X \mapsto \Phi(X) = (P[x_n], Q[x_n], \inf[x_n])$

and two metries d_1, d_2 can be defined as follows:

Definition 1.For $x \in \underset{\sim}{R}$ and $y \in \underset{\sim}{R}$, let

$d_1(x, y) = \max\{|P[x] - P[y]|, |Q[x] - Q[y]|, |\inf[x] - \inf[y]|\}$

Definition 2. For $x \in \underset{\sim}{R}$ and $y \in \underset{\sim}{R}$, let

$$d_2(x, y) = \sqrt{(P[x] - P[y])^2 + (Q[x] - Q[y])^2 + (\inf[x] - \inf[y])^2}$$

It is easy to see that $(\underset{\sim}{R}, d_1)$ and $(\underset{\sim}{R}, d_2)$ both are metric spaces and that $X_n \overset{d_1}{\mapsto} X$ iff $X_n \overset{d_2}{\mapsto} X$.

$$(\underset{\sim}{R}, d_1) \text{ or } (\underset{\sim}{R}, d_2)$$

is called the grey metric space.

About sequences of grey numbers, we have the following propositions.

Proposition 1. If $\lim\limits_{n\to\infty} x_n = a$, $\lim\limits_{n\to\infty} y_n = b$,

$$\text{then } \lim_{n\to\infty} x_n y_n = ab.$$

Proposition 2. If $\lim\limits_{n\to\infty} x_n = a$, $\lim\limits_{n\to\infty} y_n = b$, and $O \bar{\in} [P[b], Q[b]]$,

$$\text{then } \lim_{n\to\infty} \frac{x_n}{y_n} = \frac{a}{b}.$$

According to Φ, to each grey number sequence $\{X_n\}$, there correspond three real number sequences, $\{P[x_n]\}$, $\{Q[x_n]\}$, $\{\inf[x_n]\}$ and

$$\lim_{n\to\infty} x_n = a \Leftrightarrow \lim_{n\to\infty} P[x_n] = P[a],\ \lim_{n\to\infty} Q[x_n] = Q[a],\ \lim_{n\to\infty} \inf[x_n] = \inf[a].$$

Thus the convergence of a sequence of grey number is equivalent to the convergence of three sequences of real numbers.

Proof of proposition 1

1°. $\lim\limits_{n\to\infty} P[x_n y_n] = \lim\limits_{n\to\infty} \min\{P[x_n]\cdot P[y_n],\ P[x_n]\cdot Q[y_n],$

$$Q[x_n]\cdot P[y_n],\ Q[x_n]\cdot Q[y_n]\}$$

$$= \min\{\lim_{n\to\infty} P[x_n]\cdot P[y_n], \lim_{n\to\infty} P[x_n]\cdot Q[y_n],$$

$$\lim_{n\to\infty} Q[x_n]\cdot P[y_n], \lim_{n\to\infty} Q[x_n]\cdot Q[y_n]\}$$

$$= \min\{\lim_{n\to\infty} P[x_n]\cdot \lim_{n\to\infty} P[y_n], \lim_{n\to\infty} P[x_n]\cdot \lim_{n\to\infty} Q[y_n],$$

$$\lim_{n\to\infty} Q[x_n] \cdot \lim_{n\to\infty} P[y_n], \lim_{n\to\infty} Q[x_n] \cdot \lim_{n\to\infty} Q[y_n]\}$$

$$= \min\{P[a] \cdot P[b],\ P[a] \cdot Q[b],\ Q[a] \cdot P[b],\ Q[a] \cdot Q[b]\}$$

$$= P[ab].$$

2°. $$\lim_{n\to\infty} Q[x_n y_n] = \lim_{n\to\infty} \max\{P[x_n] \cdot P[y_n],\ P[x_n] \cdot Q[y_n],$$

$$Q[x_n] \cdot P[y_n],\ Q[x_n] \cdot Q[y_n]\}$$

$$= \max\{\lim_{n\to\infty} P[x_n] \cdot P[y_n], \lim_{n\to\infty} P[x_n] \cdot Q[y_n],$$

$$\lim_{n\to\infty} Q[x_n] \cdot P[y_n], \lim_{n\to\infty} Q[x_n] \cdot Q[y_n]\}$$

$$= \max\{\lim_{n\to\infty} P[x_n] \cdot \lim_{n\to\infty} P[y_n], \lim_{n\to\infty} P[x_n] \cdot \lim_{n\to\infty} Q[y_n],$$

$$\lim_{n\to\infty} Q[x_n] \cdot \lim_{n\to\infty} P[y_n], \lim_{n\to\infty} Q[x_n] \cdot \lim_{n\to\infty} Q[y_n]\}$$

$$= \max\{P[a] \cdot P[b],\ P[a] \cdot Q[b],\ Q[a] \cdot P[b],\ Q[a] \cdot Q[b]\}$$

$$= Q[ab].$$

3°. Since $\lim_{n\to\infty} \inf[x_n] = \inf[a]$, $\lim_{n\to\infty} \inf[y_n] = \inf[b]$, and for each grey number x, inf [X] takes only two volues O and 1, there exists an N such that $\inf[x_n] = \inf[a]$ and $\inf[y_n] = \inf[b]$ for every $n > N$, thus

$\inf[x_n y_n] = \inf[x_n] \cdot \inf[y_n] = \inf[a] \cdot \inf[b] = \inf[ab]$ $(n > N)$

which implies

$$\lim_{n\to\infty} \inf[x_n y_n] = \inf[ab]$$

Proof of proposition 2.

It needs only to prove the following

Lemma. If $\lim_{n\to\infty} y_n = b$ and $O \bar{\in} [P[b],\ Q[b]]$, then $\lim_{n\to\infty} \frac{1}{y_n} = \frac{1}{b}$.

Proof. We shall prove.

$\lim_{n\to\infty} P[\frac{1}{y_n}] = P[\frac{1}{b}]$, $\lim_{n\to\infty} Q[\frac{1}{y_n}] = Q[\frac{1}{b}]$, $\lim_{n\to\infty} \inf[\frac{1}{y_n}] = \inf[\frac{1}{b}]$.

Since $O \bar{\in} [P[b],\ Q[b]]$ and $\lim_{n\to\infty} P[y_n] = P[b]$, $\lim_{n\to\infty} Q[y_n] = Q[b]$, there exists an N_1 such that $O\overline{\in}[P[y_n], Q[y_n]]$ for $n > N_1$

thus by the definition of $\frac{1}{y_n}$, *we have*

$$\lim_{n\to\infty} P[\frac{1}{y_n}] = \lim_{n\to\infty} \frac{1}{Q[y_n]} = \frac{1}{Q[b]} = P[\frac{1}{b}]$$

and $$\lim_{n\to\infty} Q[\frac{1}{y_n}] = \frac{1}{P[b]} = Q[\frac{1}{b}].$$

Now $\lim_{n\to\infty} y_n = b$ *is equivalent to*

$$d_1(y_n, b) = max\{|P[y_n] - P[b]|, |Q[y_n] - Q[b]|, |inf[y_n] - inf[b]|\}$$

$\to 0$ *So there exists an* N_2 *such that*

$|inf[y_n] - inf[b]| < 1$ for $n > N_2$,

Whieh implies $inf[y] \equiv inf[b]$ for $n > N_2$, whence

$$\lim_{n\to\infty} inf[\frac{1}{y_n}] = \lim_{n\to\infty} inf[y_n] = inf[b] = inf[\frac{1}{b}]$$

References

1. Wu Heqin et al ., Introduction to Grey Mathematics, Hebei people's publishing Company, Shijia Zhuang.

2. De julung, Fundamental Metheds of Grey Systems, Huazhung Institute of Technology press, Wuhan.