SOLUTION METHOD OF SIMPLE GREY EQUTION

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0, Introduction

From the text (1), we know two finite rational grey numbers $G_{a,b.}$, $G_{c,d}$. Their sum, difference, product and quotient are:

$$G_{a,b} \oplus G_{c,d} = G_{a+c,\, b+d.}; \quad G_{a,b} \ominus G_{c,d} = G_{a-d,\, b-c.};$$

$$G_{a,b} \odot G_{c,d} = G_{e,\, f.}; \quad G_{a,b} \oslash G_{c,d} = G_{a,b} \odot G_{\frac{1}{d},\frac{1}{c}}..$$

e=min{ac,ad,bc,bd}.

f=max{ac,ad,bc,bd}.

So far as there is at leass one grey number of information type from among $G_{a,b}$ 、$G_{c,d}$. Their sum, difference, product and quotient must be the grey number of information. There are not real “negative element” and “inverse element” in the set of rational grey number.

$$“G_{a_1,b_1} \odot G_{x,y} \oplus G_{a_2,b_2} = G_{0,0}.”$$

and $$“G_{a_1,b_1} \odot G_{x,y} = G_{a_2,b_2}.”$$

are two kinds of simple grey equation.

Because there are not real “negative element” and “inverse element” in the set of rational grey number.We can't solve two kints of simple grey eguation of which we use in classical mathematics.How then? for the present, many experts and scholars are searching the way out? Now I put forward a solution of my own only for reference.

For the sake of simplicity, we convention that G_{a_1,b_1}, G_{a_2,b_2}. from the following equation represent finite rational grey numbers. The explaination of the said convention to be repeated.

I, Extract solution

Let's first discuss the solution of grey equation: "$G_{a_1,b_1} \odot G_{x,y} \oplus G_{a_2,b_2} = G_{0,0}$." ...... (1)

From the definition of addition and multiplication by rational grey number we know:

If (1) has a solution, the solution must be rational grey number.

As $G_{a_2,b_2} \neq G_{0,0}$ then (1) has no solution

As $G_{a_2,b_2} = G_{0,0}$. if $G_{a_1,b_1} \neq G_{0,0}$. (1) only one solution: $G_{0,0}$.

if $G_{a_1,b_1} = G_{0,0}$. (1) infinite solution.

Next we take up the solution of grey equation: "$G_{a_1,b_1} \odot G_{x,y} = G_{a_2,b_2}$" ......(2)

It is to be discussed under seven circumstances.

1, The two end points of grey numbers G_{a_1,b_1} and G_{a_2,b_2}. are nonnegative numbers.

According to the definition of the multiplication calculation of rational grey numbre we know that if (2) has a solution, the two end points of the solution must be nonnegative limited rational numbers.

By the definite of multiplication of rational grey number and equal concept of rational grey number, we know:

$$\begin{cases} a_1 x = a_2, \\ b_1 y = b_2. \end{cases}$$

If $a_1 \neq 0$, $b_1 \neq 0$. then $\begin{cases} x = \frac{a_2}{a_1}. \\ y = \frac{b_2}{b_1}. \end{cases}$

as $0 \leq \frac{a_2}{a_1} \leq \frac{b_2}{b_1}$ (2) has only one solution: $G_{\frac{a_2}{a_1}, \frac{b_2}{b_1}}$.

If $a = 0$ or if $b = 0$ then (2) has infinite solution or no solution.

2, The two end points of grey number G_{a_1,b_1}. and G_{a_2,b_2}. are nonpositive.

Be after the example of (1), we have

If $a_1 \neq 0$, $b_1 \neq 0$ then $\begin{cases} x = \frac{b_2}{a_1} . \\ y = \frac{a_2}{b_1} . \end{cases}$

namely (2) has only one solution: $G_{\frac{a_2}{b_1}, \frac{b_2}{a_1}}$ . .

If $a=0$ or if $b=0$ then (2) has infinite solution or no solution.

3 The two end points of G_{a_1, b_1} are nonnegative, and the two end points of G_{a_2, b_2} are nonpositive.

As above we have

$$\begin{cases} b_1 x = a_2 \\ a_1 y = b_2 \end{cases}$$

if $a_1 \neq 0$ and $b_1 \neq 0$ then $\begin{cases} x = \frac{a_2}{b_1} , \\ y = \frac{b_2}{a_1} , \end{cases}$. namely (2) has a only one solution: $G_{\frac{a_2}{b_1}, \frac{b_2}{a_1}}$.

if $a_1 = 0$ and $b_1 = 0$. then (2) has infinite solution or no solution. .

4, The two end points of G_{a_1, b_1} are nonposition. The two end points of G_{a_2, b_2} are nonnegative.

In the same manner we know : $\begin{cases} a_1 x = b_2 , \\ b_1 y = a_2 . \end{cases}$

If $a_1 \neq 0$ and $b_1 \neq 0$. then $\begin{cases} x = \frac{b_2}{a_1} \\ y = \frac{a_2}{b_1} \end{cases}$ namely (2) has only one solution: $G_{\frac{b_2}{a_1}, \frac{a_2}{b_1}}$.

If $a_1 \neq 0$ and if $b_1 = 0$. then (2) has infinite solution or no solution.

5, The two end points of G_{a_1, b_1} : $a_1 \leq 0 \leq b_1$. The two end points of G_{a_2, b_2} are at the same time nonpositive or nonnegative.

By multiplication of rational grey number we know, (2) has no solution.

6, The two end points of G_{a_1, b_1}. : $a_1 \leq 0 \leq b_1$.

The two end points of $G_{a_2 . b_2}$: $a_1 \leq 0 \leq b_2$.

By multiplication of rational grey number we know: The two end points of unknown rational grey number $G_{x,y}$ has three circumstance as following

① $0 \leq x \leq y$, ② $x \leq y \leq 0$. ③ $x \leq 0 \leq y$.

We know (2) has infinite solution under circumstances of ① and ② .

Now we take up the third circumstance, namely $x \leq 0 \leq y$. Under this circumstance, as $|a_1| \geq b_1$

By the definite of multiplication we know:

if (2) has definition of solution ,then $\begin{cases} x_1 = \frac{b_2}{a_1}. \\ y_1 = \frac{a_2}{b_1}. \end{cases}$

As $|a_1| \leq b_1$

in the same manner ,we know:

If (2) has definite solution then $\begin{cases} x_1 = \frac{a_2}{b_1}. \\ y_1 = \frac{b_2}{b_1}. \end{cases}$ if not , (2) has infinite solution or no solution.

7, The two end points of G_{a_1,b_1}. are at the same time nonpositive or nonnegative. The two end points of G_{a_2,b_2}.: $a_2 \leq 0 \leq b_2$.

By definition of multiplication we know, the two end points of $G_{x,y}$: $x \leq 0 \leq y$.

~~x~~ as a_1, b_1 are at the same time nonnegative,then $\begin{cases} b_1 x_1 = a_2 \\ b_1 y_1 = b_2 \end{cases}$

as $b_1 \neq 0$ then $\begin{cases} x_1 = \frac{a_2}{b_1}, \\ y_1 = \frac{b_2}{b_1}, \end{cases}$

as a_1, b_1 are at the same time nonpositive , in the same manner we know:

$$\begin{cases} a_1 y_1 = a_2 \\ a_1 x_1 = b_2 \end{cases} \quad \text{as} \quad a_1 \neq 0 \quad \text{then} \quad \begin{cases} x_1 = \frac{b_2}{a_1} \\ y_1 = \frac{a_2}{a_1} \end{cases}$$

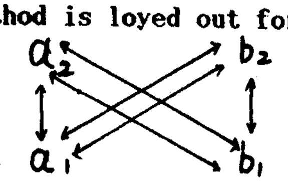
if not , (2) has infinite solution or no solution.

Mentioned above, if grey equation : "$G_{a_1,b_1} \odot G_{x,y} = G_{a_2 b_2}$" has definite solution , its limits be among the finite rational grey numbers with the following four groups of numbers as their end points .

$$\frac{a_2}{a_1} 、 \frac{b_2}{b_1} ; \quad \frac{a_2}{a_1} 、 \frac{b_2}{a_1} ;$$

$$\frac{a_2}{b_1} 、 \frac{b_2}{a_1} ; \quad \frac{b_2}{b_1} , \frac{a_2}{b_1}$$

The following method is loyed out for the sake of easy memory.



Namely: The left ratio, the right ratio; The two opposite angles ratio; The left ratio, the opposite angle ratio; The right ratio, the opposite angle ratio.

II, For example

According to above we know:

Normol way of solving to equation: "$G_{a_1,b_1} \odot G_{x,y} = G_{a_2,b_2}$" is:

1, Work out four groups of number with the above method.

2, Define the solution of the equation from among the limits.

3, Check: The solutions are substituted in original equation, we have all solutions.

Example 1,

Solve grey equation: $G_{-2,-1} \odot G_{x,y} = G_{3,8}$.

Solve: since $a_2=3$, $b_2=8$, $a_1=-2$, $b_1=-1$.

Then the four groups of number are:

$-\frac{3}{2}$, -8; -3, -4; $-\frac{3}{2}$, -4; -8, -3.

Then the solution limits of the original equation are:

$G_{-8,-\frac{3}{2}}$; $G_{-4,-3}$; $G_{-4,-\frac{3}{2}}$; $G_{-8,-3}$.

Substituting the above solution limits in the original equation respectively, the solution of original eguation is $G_{-4,-3}$.

Example 2,

Solve equation: $G_{1,2} \odot G_{x,y} = G_{-7,-2}$.

Solve: Since $a_2=-7$, $b_2=-2$, $a_1=1$, $b_1=2$.

There four groups of numbers: -7, -1; -2, $-\frac{7}{2}$; -7, -2; -1, $-\frac{7}{2}$.

Then the solution limits of the original equation are:

$G_{-7,-1}$; $G_{-\frac{7}{2},-2}$; $G_{-7,-2}$; $G_{-\frac{7}{2},-1}$.

Substituting the above solution limits in the original equation respectively, the solution of original equation is $G_{-\frac{7}{2},-1}$.

Example 3,

Solve grey equation: $G_{0,5} \odot G_{x,y} = G_{2,7}$.

Solve: Since $a_2=2$, $b_2=7$, $a_1=0$, $b_1=5$.

There is one groups of number in the four groups of number: $\frac{7}{5}, \frac{2}{5}$.

Then the solution limits of the original equation is: $G_{\frac{2}{5}, \frac{7}{5}}$.

Apparenty, $G_{\frac{2}{5}, \frac{7}{5}}$ isn't the solution of the original equation. Then the original equation has infinite solution or no solution.

Example 4,

Solve grey equation: $G_{-8,1} \odot G_{x,y} = G_{-4,2}$.

Solve : Since $a_2 = -4$, $b_2 = 2$, $a_1 = -8$, $b_1 = 1$

There are four groups : $\frac{1}{2}, 2$; $-\frac{1}{4}, -4$; $\frac{1}{2}, -\frac{1}{4}$; $2, -4$.

Then the limits of original equation's solution are: $G_{\frac{1}{2},2}$; $G_{-4,\frac{1}{4}}$; $G_{-\frac{1}{4},-\frac{1}{2}}$; $G_{-4,2}$.

Substitute original equation cheking: It is no solution.

Then original equation has infinite solution or no solution.

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