THE ALGEBRAIC OPERATION AND PROPERTIES OF GREY SUBSETS

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Based on the paper , We discuss the algebraic operation and properties of grey subsets.

I, The algebraic operation of grey subsets.

Definition 1, Let A, B be independent grey subsets on the universe U, The algebraic product of A and B is expressed by A·B

$$\overline{\mu}_{A\cdot B} \triangleq \overline{\mu}_A \cdot \overline{\mu}_B$$

$$\underline{\mu}_{A\cdot B} \triangleq \underline{\mu}_A \cdot \underline{\mu}_B$$

evidently, $0 \leq \underline{\mu}_{A\cdot B} \leq \overline{\mu}_{A\cdot B} \leq 1$.

Definition 2, Let A, B be independent grey subsets on the universe U, the general algebraic Sum of A and B is expressed by A⊕B

$$\overline{\mu}_{A\oplus B} \triangleq \overline{\mu}_A + \overline{\mu}_B - \overline{\mu}_A \cdot \overline{\mu}_B$$

$$\underline{\mu}_{A\oplus B} \triangleq \underline{\mu}_A + \underline{\mu}_B - \underline{\mu}_A \cdot \underline{\mu}_B$$

evidently, $0 \leq \underline{\mu}_{A\oplus B} \leq \overline{\mu}_{A\oplus B} \leq 1$.

Definition 3, Let A , B be independent grey subsets on the universe U and $\bar{\mu}_A + \bar{\mu}_B \leq 1$. The narrow algebraic Sum of A and B is expressed by A+B,

$$\bar{\mu}_{A+B} \triangleq \bar{\mu}_A + \bar{\mu}_B$$

$$\underline{\mu}_{A+B} \triangleq \underline{\mu}_A + \underline{\mu}_B$$

Definition 4, Let A, B be independent grey subsets on the universe U, the absolute difference of A and B is expressed by $|A-B|$

$$\bar{\mu}_{|A-B|} \triangleq max\{|\bar{\mu}_A - \bar{\mu}_B|, |\underline{\mu}_A - \underline{\mu}_B|\}$$

$$\underline{\mu}_{|A-B|} \triangleq min\{|\underline{\mu}_A - \underline{\mu}_B|, |\bar{\mu}_A - \bar{\mu}_B|\}$$

Evidently, When grey subsets A, B are turned into fuzzy subsets, the algebraic product, sum and absolute difference of grey subsets are regarded as those of fuzzy subsets. In this sense, algebraic operation of grey subsets is the generalization of algebraic

operation of fuzzy subsets while fuzzy' operation is a partcular example of grey' operation.

II, The properties of the algebraic operation of grey subsets

Baset on the algebraic operation of grey subsets. We discuss properties of the algebraic operation of grey subsets.

Property 1, If A, B and C are independent grey subsets, then

(1), A·B = B·A

(2) (A·B)·C = A·(B·C)

Proof:

since $\bar{\mu}_{A\cdot B} = \bar{\mu}_A \cdot \bar{\mu}_B = \bar{\mu}_B \cdot \bar{\mu}_A = \bar{\mu}_{B\cdot A}$

$$\underline{\mu}_{A\cdot B} = \underline{\mu}_A \cdot \underline{\mu}_B = \underline{\mu}_B \cdot \underline{\mu}_A = \underline{\mu}_{B\cdot A}$$

hence $A \cdot B = B \cdot A$.

since
$$
\begin{aligned}
\bar{\mu}_{(A\cdot B)\cdot C} &= \bar{\mu}_{A\cdot B}\cdot\bar{\mu}_{C} \\
&= \bar{\mu}_{A}\cdot\bar{\mu}_{B}\cdot\bar{\mu}_{C} \\
&= \bar{\mu}_{A}\cdot\bar{\mu}_{B\cdot C} \\
&= \bar{\mu}_{A\cdot(B\cdot C)}
\end{aligned}
$$
$$
\begin{aligned}
\underline{\mu}_{(A\cdot B)\cdot C} &= \underline{\mu}_{A\cdot B}\cdot\underline{\mu}_{C} \\
&= \underline{\mu}_{A}\cdot\underline{\mu}_{B}\cdot\underline{\mu}_{C} \\
&= \underline{\mu}_{A}\cdot\underline{\mu}_{B\cdot C} \\
&= \underline{\mu}_{A\cdot(B\cdot C)}
\end{aligned}
$$

hence, $(A\cdot B)\cdot C = A\cdot(B\cdot C)$

Property 2, If A,B are two grey subsets, then

$$A \oplus B = (A^{c}\cdot B^{c})^{c}$$

Proof:
$$
\begin{aligned}
\bar{\mu}_{A\oplus B} &= \bar{\mu}_{A}+\bar{\mu}_{B}-\bar{\mu}_{A}\cdot\bar{\mu}_{B} \\
&= 1-(1-\bar{\mu}_{A})(1-\bar{\mu}_{B}) \\
&= 1-\underline{\mu}_{A^{c}}\cdot\underline{\mu}_{B^{c}} \\
&= 1-\underline{\mu}_{A^{c}\cdot B^{c}} \\
&= \bar{\mu}_{(A^{c}\cdot B^{c})^{c}}
\end{aligned}
$$

According to the same method we can infer:

$$\underline{\mu}_{A\oplus B}=\underline{\mu}_{(A^c\cdot B^c)^c}$$

hence $$A\oplus B=(A^c\cdot B^c)^c$$

Property 3, If A, B and C are independent grey subsets, and $\bar{\mu}_A+\bar{\mu}_B\leq 1$ then

(1) A+B = B+A

(2) (A+B)·C = A·C + B·C

Proof: (1) $$\bar{\mu}_{A+B}=\bar{\mu}_A+\bar{\mu}_B$$
$$=\bar{\mu}_B+\bar{\mu}_A$$
$$=\bar{\mu}_{B+A}$$

According to the same method we can infer: $\underline{\mu}_{A+B}=\underline{\mu}_{B+A}$

hence A+B = B+A.

(2) since $\bar{\mu}_{A\cdot C}+\bar{\mu}_{B\cdot C}=(\bar{\mu}_A+\bar{\mu}_B)\cdot\bar{\mu}_C\leq 1$

hence A·C + B·C is effective.

$$\bar{\mu}_{A\cdot C}+\bar{\mu}_{B\cdot C}=(\bar{\mu}_A+\bar{\mu}_B)\cdot\bar{\mu}_C$$
$$=\bar{\mu}_{A+B}\cdot\bar{\mu}_C$$
$$=\bar{\mu}_{(A+B)\cdot C}$$

i.e. $$\bar{\mu}_{(A\cdot C+B\cdot C)}=\bar{\mu}_{A\cdot C}+\bar{\mu}_{B\cdot C}$$
$$=\bar{\mu}_{(A+B)\cdot C}$$

According to same method we can infer:

$$\underline{\mu}_{(A\cdot C+B\cdot C)}=\underline{\mu}_{A\cdot C}+\underline{\mu}_{B\cdot C}$$
$$=\underline{\mu}_{(A+B)\cdot C}$$

hence , $(A+B)\cdot C = A\cdot C + B\cdot C$.

Property 4, If A, B are independent grey subsets, then

$$|A-B|=|B-A|$$

Proof: (omitted)

Evidently, When A, B and C are all turned into fuzzy subsets, above four properties of grey subsets are will become properties of fuzzy subsets.

Hence, the algebraic operation of grey subsets is the generalization of algebraic operation of fuzzy subsets.

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