II TYPE EQUATION OF A FUZZY MATRIX (I)

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ABSTRACT

In this paper we presented a definition of the II type equation of a fuzzy matrix, and we discussed preliminarily its properties.

Keyword: II type equation of a fuzzy matrix.

1. PREFACE

Liu Wangjin [1] introduced the concepts of the realizable problem of a fuzzy symmetric matrix and its content. And many estimated formulas of the content of a fuzzy symmetric matrix are gave [see 1—5] . But so far, the people can not directly compute it. When the content of a fuzzy matrix is exist, we may compute directly the content of the fuzzy matrix by the II type equation of the fuzzy matrix.

2. II TYPE EQUATION OF A FUZZY MATRIX

In the paper [6] we were define fuzzy relational N-D equation of a fuzzy matrix. Here we shall define II type equation of a fuzzy matrix.

Definition 2.1 Let $B\in\mathcal{M}_{n\times n}$ is a non-zero fuzzy matrix. A equation in the form of

$$B_{n\times n} = X_{n\times t}\, X'_{t\times n} \qquad (2.1)$$

is called a fuzzy relational non-deterministic equation of II type of B, or a N-D equation of II type of B, and or a II type equation of B. (Where B is known, and X is unknown). The t is called its index. The $X_{n\times t}$ such that (2.1) holds is called the solution of the equation when the index is t.

The following properties on the II type equation are easily proved:

Theorem 2.1 Let for an index t, the II type equation of $B\in\mathcal{M}_{n\times n}$ has a solution $A = (a_{ij})_{n\times t}$. We write a=

$\min_{i,j}\{a_{ij}\}$. And if some columns are increased in the A, all increasing elements are no greater than the a, and we shall get a fuzzy matrix A^*. Then the A^* is also a solution of the equation.

Theorem 2.2 Let B_1, $B_2 \in \mathcal{M}_{n\times n}$, and the II type equation of B_1 and B_2 have a solution when an index is t. Then the II type equation of $B_1 + B_2$ has also a solution when the index is t.

Theorem 2.3 Let the II type equation of $B \in \mathcal{M}_{n\times n}$ has a solution when an index is t. Then the II type equation of kB has also a solution when the index is t, where $k\in[0,1]$.

Theorem 2.4 Let the II type equation of $B\in\mathcal{M}_{n\times n}$ has a solution $A_{n\times t}$. And if the row p and the row q of the B are exchnged, at the same time the column p and the column q of the B are exchnged, and we shall get B^*. Then the II type equation of B^* has a solution A^* when the index is t. where we shall get A^* if the row p and the row q of the A are exchenged.

Theorem 2.5 Let the II type equation of $B = (b_{ij})_{n\times n}$ has a solution when an index is t. Then II type equation of

$$B = \begin{pmatrix} b_{11} & \cdots & b_{1j} \\ \cdots & \cdots & \cdots \\ b_{j1} & \cdots & b_{jj} \end{pmatrix} \quad (j=1,2,\ldots,n)$$

has also a solution when the index is t.

Theorem 2.6 Let the II type equation of $B\in\mathcal{M}_{n\times n}$ has a solution when an index is t. The some rows of the B are deleted, at the same time corresponding columns of the B are also deleted .We shall get B^*. Then the II type equation of B^* has also a solution when the index is t.

Theorem 2.7 Let the II type equation of $B\in\mathcal{M}_{n\times n}$ has a solution when an index is t . We insert a row of the same elements in the B, at the same time insert the corrsponding column of the same elements in the B. We shall get B^*. Then the II type equation of the B^* has also a solution when the index is t.

Theorem 2.8 Let the II type equation of $B\in\mathcal{M}_{n\times n}$ has not any solution when an index is $t = s$. Then II type equation of B has not any solution when the index $t < s$.

Theorem 2.9 Let the II type equation of $B\in\mathcal{M}_{n\times n}$ has a solution A when an index is t. We transpose two columns of the A, and get a matrix A^*. Then the A^* is a solution of the II type equation.

Theorem 2.10 If the II type equation of $B \in \mathcal{M}_{n \times n}$ has a solution when an index is t. Then B is symmetrical.

Definition 2.2 Let $B \in \mathcal{M}_{n \times n}$ is symmetrical. If the II type equation of the B has a solution when an index is t. We are called the II type equation of B has a solution.

Theorem 2.11 The set of all $n \times n$ fuzzy matrices such that the II type equations of these fuzzy matrices are solution forms a fuzzy linear space.

Definition 2.3 Let $B \in \mathcal{M}_{n \times n}$ is a symmtrical, and let
$\beta(B) = \min\{t \mid \text{the II type equation of B has a solution}\}$.
Then the $\beta(B)$ is called the minimum index when the II type equation of the B has a solution.

Obviously if the II type equation of $B \in \mathcal{M}_{n \times n}$ has a solution, then $\beta(B) > 0$. If it has not any solution, then $\beta(B)=0$.

Definition 2.4 [1] Let $B \in \mathcal{M}_{n \times n}$ is symmetrical. B is said to be realizable if there is $A \in \mathcal{M}_{n \times n}$ such that $B = A A'$. The A is said to be a realization of B.

And let
$\mu(B) = \min\{m \mid \exists A \in \mathcal{M}_{n \times m}, \text{ such that } B = A A'\}$,
the $\mu(B)$ is called the content of B.

We easy see that for arbitrary fuzzy symmtrical matrix B have $\beta(B) = \mu(B)$. Therefore we may compute directly the content of the fuzzy matrix by the II type equation of the fuzzy matrix.

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