A CRITICAL APPRECIATION ON FUZZY LOGIC CONTROLLER

Part I

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Abstract

In this paper, without denying the merits and novelty of the technique of approximate ressoning as presented by Zadeh [1] , we are proposing an alternative method for inexact ressoning that is simple to understand and implement. The proposed method desires an elementary knowledge in curve-fitting, a well-posed problem. It has been established that the proposed technique gives at least the same, if not better in all cases, results as found from the application of Zadeh's compositional rule of inference. The proposed method of inexact reasoning is first described and then illustrated with examples. The new results are compared with those obtained by the application of the existing rules of approximate reasoning. Ultimately we indicate the uselessness of the application of approximate reasoning in the design of fuzzy logic controller.

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I. Introduction

The celebrated concept of approximate reasoning has been tremendously used in the design of fuzzy logic controller [4,5,6,7,8,9,10,11, 12,13,14]. The success of the fuzzy logic controller, in a very complex situation, has been claimed by many researchers. Hence it has got a wider field of applications in process control. There are many complex situations whose dynamics cannot be modeled appropriately in terms of state-space or transfer-function. Under such circumstances the ultimate controller design (conventional controller) which is based upon the analysis of the model (state space/ transfer-function) cannot be appropriate. Therefore, after the implementation of such controller in real plant system, we have to go for a trial and error process to readjust the controller parameters with the real dynamics of the situation. But the control laws of such complex situation can be well captured from the intuition and experience of an expert operator who makes linguistic statements/instructions (using linguistic variables viz. pressure is big, temperature is high, current is low etc.) about the situation to be controlled. Even if the input/output parameters are precisely measurable, still we can use fuzzy logic controller [4,5,6,7,8,9,10,11,12,1,3,14] to control a complex plant/system. In this context, the intention of fuzzy logic is to deal with the phenomenon of linguistic vagueness [2] . The output of the fuzzy logic controller is ultimately defuzzified [3,4,5,11,12,13,14] and injected to plant/system as a control action. Thus a non-fuzzy output of a plant/system is ultimately controlled by non-fuzzy input to the plant/

system. But the inbetween decisionmaking of the controller is performed using fuzzy logic because of the vagueness of the linguistic statement/ instruction of an expert operator.

In this paper our basic aim is to demonstrate that fuzzy quantification (in terms of membership function) of the perception [1] of an expert operator is not essential to capture the control laws of a very complex situation which cannot be modeled appropriately in conventional form (state space/transfer function). Instead of forming a fuzzy relational matrix R from

$$\text{if } X \text{ is } A \text{ then } Y \text{ is } B$$

type statements where X and Y are two linguistic variables and A,B are two fuzzy subsets over their respective universe of discourses U and V we consider a simple conventional relation of the form

$$y = f(x)$$

where x,y are two real variables which takes on values in U and V respectively. It is found that the above relation can also capture the control laws derived from the experience and intuition of an expert operator. Thus we can ignore the unnecessary fuzzification and defuzzification of the input and output variables of the fuzzy logic controller and avoid the construction of controller, unnecessarily through fuzzy logic which has a tremendous application in many other fields of science and engineering.

II. Formulation of the problem

Let X and Y represent two linguistic variables taking values in U and V respectively and let p and q be two typical premises expressed in natural language. Also let p and q translates into the possibility assignment equations

$$p \Leftrightarrow X \text{ is } H \rightarrow \Pi_X = H$$

and $q \Leftrightarrow$ if X is F then Y is G $\rightarrow \Pi_{(X,Y)} = R$. Here the symbol $\Leftrightarrow$ stands for " is defined to be"; F,H are fuzzy subsets of U; G is a fuzzy subset of V and the relation R is a fuzzy subset of the cartesian product UxV.

Then we infer

$$r \leftarrow \Pi_Y = H \circ R$$

where o denotes the well-known composition as defined by Zadeh [1], viz.

$$\Pi_Y = \Pi_X \circ \Pi_{(X,Y)}$$

and

Here we have considered the linguistic variables X and Y in the premises p and q range over finite sets or can be approximated by variables ranging over such sets [1] in order that F,G,H and R may be represented by finite-dimensional relational matrices. Again the relation R can be formed in many different ways using different translation rules for compound assertions. Here, in this paper, we shall consider only two such widely used translation rules [1] viz.

i) if X is F then Y is G -> $\Pi_{(X,Y)} = \bar{F}' \oplus \bar{G}$

ii) if X is F then Y is G -> $\Pi_{(X,Y)} = \bar{F} \cap \bar{G} = F \times G$

where $\Pi_{(X,Y)}$ denotes the possibility distribution of the binary variable (X,Y), $\bar{F}$ and $\bar{G}$ are the cylindrical extensions of F and G respectively, that is,

$$\bar{F} = F \times V$$

and $$\bar{G} = U \times G \; ;$$

FxG is the cartesian product of F and G, which may be expressed as $\bar{F} \cap \bar{G}$ and is defined by

$$\mu_{F \times G}(u,v) = \mu_F(u) \wedge \mu_G(v) \; : \; u \in U, \; v \in V,$$

F' denotes the complement of F and the bounded sum $\oplus$ is defined by

$$\mu_{\bar{F}' \oplus \bar{G}}(u,v) = 1 \wedge [1 - \mu_F(u) + \mu_G(v)]$$

where + and - denote arithmetic sum and difference and $\wedge$ denote minimum.

Now every relational matrix R of the form, discussed so far, when defuzzified defines an elementwise relation between two distinct variables say, current and rotational speed of a motor and gives us informations about variations of a single variable with the change of the independent variable. Thus we can have a collection of a pair of observations of the two variables from the above relation R. Letting them as points in a two-dimensional plane we can always fit an interpolation polynomial that is defined over a large domain containing UxV. For our present discussion let x and y be two classical variables which takes on values in U' and V' where U' and V' are two continuums containing U and V respectively. And let f : U' -> V' be a function generated from the relational matrix R. The problem of approximate reasoning then reduces to finding a value of y corresponding to a particular value of x. This particular value of x is found from the defuzzification [3] of the premise

$$p \Leftrightarrow X \text{ is } H.$$

Also it is found that the value of y as found from the proposed technique is sharper than the value of y as found from the defuzzification of the relational matrix

$$\Pi_Y = \Pi_X \circ \Pi_{(X,Y)} .$$

So far we have considered simple inference from a fact and a rule both containing inexact concepts. Let's consider the case where we have to infer from a single fact of the form

$$p \Leftrightarrow X \text{ is } H$$

and a set of rules of the form

$$q \Leftrightarrow \text{if } X \text{ is } F \text{ then } Y \text{ is } G$$

viz.

$$q_1 \Leftrightarrow \text{if } X \text{ is } F_1 \text{ then } Y \text{ is } G_1$$

$$q_2 \Leftrightarrow \text{if } X \text{ is } F_2 \text{ then } Y \text{ is } G_2$$

....

$$q_n \Leftrightarrow \text{if } X \text{ is } F_n \text{ then } Y \text{ is } G_n$$

where $n < \infty$.

For problems of this type we first find a unified relational matrix R for inference. For this, let $p, q_1, q_2, \ldots, q_n$ translates into the possibility assignment equations

$$p \Leftrightarrow \Pi_X = H$$

$$q_1 \Leftrightarrow \Pi_{(X,Y)} = \bar{F}'_1 \oplus \bar{G}_1 = R_1$$

$$q_2 \Leftrightarrow \Pi_{(X,Y)} = \bar{F}'_2 \oplus \bar{G}_2 = R_2$$

$$\cdots\cdots\cdots\cdots$$

$$q_n \Leftrightarrow \Pi_{(X,Y)} = \bar{F}'_n \oplus G_n = R_n \quad .$$

Here $H, F_1, F_2, \ldots, F_n$ are fuzzy subsets of U and $G_1, G_2, \ldots, G_n$ are fuzzy subsets of V. $R_1, R_2, \ldots, R_n$ are n fuzzy subsets of UxV. Now we construct a relation R from $R_1, R_2, \ldots, R_n$ as

where

$$\mu_R(u,v) = \min\left\{\mu_{R_1}(u,v),\ \mu_{R_2}(u,v), \ldots,\ \mu_{R_n}(u,v)\right\} .$$

Then from

$$p \Leftrightarrow \Pi_X = H$$

and R we infer

$$r \leftarrow \Pi_Y = \Pi_X \circ \Pi_{(X,Y)} = H \circ R.$$

Now, for problems of this type, let's first of all, defuzzify the compound asertions $q_1, q_2, \ldots, q_n$ and collect a set of pairs of observations of the variables X and Y and generate the relation $f : U' \rightarrow V'$. Then from the defuzzification of

$$p \Leftrightarrow \Pi_X = H$$

the inference follows at once.

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