Abstract

Fuzzy set covering problem

The classical set covering problem from the discrete optimization is generalized for fuzzy sets.

Suppose that $M \equiv \{1, \ldots, m\}$, $N \equiv \{1, \ldots, n\}$, are given finite sets and $S_j: M \rightarrow [0,1]$, $j = 1, \ldots, n$, are membership functions of n given fuzzy sets, the support of which is a subset of M.

We want to construct a fuzzy system of sets $S_1, \ldots, S_n$ given by a membership function $X: N \rightarrow [0,1]$ so that the resulting system covers the set M in such a way that the element $i \in M$ belongs to the union of the subsets S_j from the fuzzy system X with the membership level greater or equal to b_i, $b_i \in [0,1]$, i.e. We want to determine the values $X(j) \in [0,1]$ for $j \in N$ satistying the relations

$$\max_{j \in N} \min \left(S_j(i), X(j)\right) = b_i \text{ for } i \in M, \quad (1)$$

$$X(j) \in [0,1] \text{ for } j \in N \quad (2)$$

A fuzzy system given by a membership function X satisfying these conditions will be called a feasible system. We are looking for a feasible system, which minimizes (or maximizes) a given objective function $f\left(X(1), \ldots, X(n)\right)$, i.e. which solves the optimization problem:

$$\left.\begin{array}{l} f\left(X(1), \ldots, X(n)\right) \rightarrow \min\ (\max) \\ \text{subject to conditions (1), (2).} \end{array}\right\} \quad (P)$$

Optimization problems of the form (P) with the following objective function were considered:

$$f\left(X(1), \ldots, X(n)\right) = \sum_{j=1}^{n} X(j) \qquad (3)$$

$$f\left(X(1), \ldots, X(n)\right) = \max_{1 \le j \le n} X(j) \qquad (4)$$

Special case, in which $X(j) = 0$ or 1 for all $j \in N$ (i.e. the system of fuzzy sets $S_1, \ldots, S_n$ is a classical nonfuzzy set so that S_j either belongs to the system or not for all $j \in N$) was investigated. If the sets S_j, $j \in N$, are classical sets too (i.e. $S_j: M \to \{0,1\}$ for all $j \in N$), the problem (P) with the objective function given by (3) reduces to the classical set covering problem from the discrete optimization.

Methods for solving the problem (P) with objective functions (3), (4) were suggested, their complexity was investigated. Other generalizations and extensions of the problem (P) were suggested, possible applications were briefly discussed.

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