GENERALIZED DETERMINISTIC, DISCRETE STOCHASTIC AND FUZZY PETRI NETS

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Abstract. The paper deals with generalized Petri net as flexibile formal means for analysis discrete systems. On the basis of generalized Petri net we are able to define any deterministic and non-deterministic (discrete stochastic and fuzzy) Petri nets.

Keywords. Generalized Petri net, deterministic Petri net, non-deterministic Petri net, enabled transition, firing of transition.

Deterministic Petri nets [1,2,3,4] and non-deterministic [5,6,7,8,9,10] Petri nets are formal graph means for property description and analysis of discrete systems. They can be generalized and represented by one Petri net.

Generelized Petri net (GPN) is a bichromatic oriented graph defined as a 5-tuple [9]:

$$GPN = \langle \mathbf{P},\ \mathbf{T},\ X,\ Y,\ Z \rangle \tag{1}$$

where :- $\mathbf{P}$ is a finite set of vertices called places, represented by circles , $\mathbf{P} = \{p_1, p_2, \ldots, p_k\}$,

- $\mathbf{T}$ is a finite set of vertices called transitions, represented by lines, $\mathbf{T} = \{t_1, t_2, \ldots, t_r\}$, $\mathbf{P} \cap \mathbf{T} = \emptyset$,
- X is an ordered n-tuple that defines the qualities of k places $\mathbf{P}$,
- Y is an ordered n-tuple that defines the qualities

of r transitions **T**,

- Z is an ordered n-tuple that defines the qualities of edges which are given by forward and backward incidence functions [1,2,3,4].

An ordered n-tuple that defines the qualities of k places **P** can be defined as follows [9]:

$$X = \langle \mathbf{C},\ IC,\ M_{\emptyset}C,\ \mathbf{UP} \rangle \quad (2)$$

where:
- **C** is a finite set of used colors (types of tokens) $\mathbf{C} = \{c_1, c_2, \dots, c_h\}$,
- IC: $\mathbf{P} \times \mathbf{T} \rightarrow \mathbf{N} \times \mathbf{C}$, ($\mathbf{N} = \emptyset, 1, \dots$), $(IC((n,c)_{m,i,j}))$, where $m \in \langle 1,h \rangle$, $i \in \langle 1,k \rangle$, $j \in \langle 1,r \rangle$, is the forward incidence function. It states for each edges m ordered 2-tuple (n,c), from place $p_i \in \mathbf{P}$ to transition $t_j \in \mathbf{T}$, where $n \in \mathbf{N}$, $c \in \mathbf{C}$. Through edge from place $p_i \in \mathbf{P}$ to transition $t_j \in \mathbf{T}$ n_m tokens of color c_m, can pass,
- $M_{\emptyset}C$: $\mathbf{P} \rightarrow \mathbf{N} \times \mathbf{C}$ is an initial marking, $(M_{\emptyset}C((n,c)_{m,i}))$, where $m \in \langle 1,h \rangle$, $i \in \langle 1,k \rangle$. It states for each place $p_i \in \mathbf{P}$, $i \in \langle 1,k \rangle$ m ordered 2-tuples (n,c), $n \in \mathbf{N}$, $c \in \mathbf{C}$, which represent, how many tokens of the given color occur in place $p_i \in \mathbf{P}$,
- $\mathbf{UP} = \{up_1, up_2, \dots, up_k\}$ is a finite set of qualities of tokens in the places $p_i \in \mathbf{P}$, which can be deterministic, stochastic or fuzzy.

An ordered n-tuple that define the qualities of r transitions **T** can be defined as follows [9]:

$$Y = \langle QC,\ \mathbf{\tau},\ \mathbf{PR},\ \mathbf{UT} \rangle \quad (3)$$

where:
- QC: $\mathbf{T} \times \mathbf{P} \rightarrow \mathbf{N} \times \mathbf{C}$, $(QC((n,c)_{m,i,j}))$, where $m \in \langle 1,h \rangle$, $i \in \langle 1,k \rangle$, $j \in \langle 1,r \rangle$ is a backward incidence function. It states for each edge m ordered 2-tuple (n,c) from transition $t_j \in \mathbf{T}$ to place $p_i \in \mathbf{P}$, where

$n \in \mathbf{N}$, $c \in \mathbf{C}$. Through edge from transition $t_j \in \mathbf{T}$ to place $p_i \in \mathbf{P}$ n_m tokens of color c_m, can pass,

- $\mathbf{\tau} = \{\tau_1, \tau_2, \dots, \tau_r\}$ is finite set of times of firings of r transitions $\mathbf{T}$,
- $\mathbf{PR} = \{pr_1, pr_2, \dots, pr_r\}$ is a finite set of predicates, which are assigned to transitions $t_j \in \mathbf{T}$,
- $\mathbf{UT} = \{ut_1, ut_2, \dots, ut_r\}$ is finite set of qualities of transitions $t_j \in \mathbf{T}$, which can be that deterministic, stochastic or fuzzy.

The finite set of qualities of edges, which is given by forward and backward incidence function, can be defined as follows [9]:

$$Z = \langle \mathbf{I}, \mathbf{E}, \mathbf{L} \rangle \tag{4}$$

where:- $\mathbf{I} = \{i_1, i_2, \dots, i_{s1}\}$ is a finite set of inhibit edges,

- $\mathbf{E} = \{e_1, e_2, \dots, e_{s2}\}$ is a finite set of empty edges,
- $\mathbf{L} = \{l_1, l_2, \dots, l_{s3}\}$ is a finite set of logical edges.

Further it holds:

$$\mathbf{I} \cap \mathbf{E} = \mathbf{I} \cap \mathbf{L} = \mathbf{E} \cap \mathbf{L} = \emptyset \tag{5}$$

where each edge is a member only of one set from sets

I or **E** or **L**.

The place $p_i \in \mathbf{P}$ is an input place of transition $t_j \in \mathbf{T}$, if in $IC(p_i, t_j)$ there exists h ordered 2-tuples (n_m, c_m), where $m \in \langle 1, h \rangle$, $\exists$ n, for which $n \geq 1$ holds. Then $p_i \in \mathbf{IN}(t_j)$, where $\mathbf{IN}(t_j)$ is set of input places of transition $t_j \in \mathbf{T}$.

The place $p_i \in \mathbf{P}$ is an output place of transition $t_j \in \mathbf{T}$, if in $QC(t_j, p_i)$ there exists h ordered 2-tuples (n_m, c_m), where $m \in \langle 1, h \rangle$, $\exists$ n, for which $n \geq 1$ holds. Then $p_i \in \mathbf{OUT}(t_j)$, where $\mathbf{OUT}(t_j)$ is set of output places of transition $t_j \in \mathbf{T}$.

Each edge is described by an ordered 2-tuple (p_i, t_j) (input edges), or (t_j, p_i) (output edges). The set of output edges of transition $t_j \in \mathbf{INN}(t_j)$ are those edges (p_i, t_j), for which $p_i \in IM(t_j)$ holds.

The transition $t_j \in T$ is enabled from marking $M_aC(p_i)$ ($a \in N$, $i \in \langle 1,k \rangle$), if for each $p \in IM(t_j)$, $m \in \langle 1,h \rangle$, $i \in \langle 1,k \rangle$ holds:

$$FIRE(t_j) = ((\prod_{b=1}^{3} FIRE_b(t_j)) \text{ AND } ((p,t_j) \in IMM(t_j)) \text{ AND}((p,t_j) \in L))$$

$$OR(not(\prod_{b=1}^{3} FIRE_b(t_j)) \text{ AND } ((p,t_j) \in IMM(t_j)) \text{ AND}((p,t_j) \in I)) \quad (6)$$

where: $FIRE_1(t_j) = (M_aC((n,c)_{m,i}) \geq IC((n,c)_{m,i,j}))$,

$FIRE_2(t_j)$ = (expired time for firing of transition t_j)),

$FIRE_3(t_j) = pr_j)$.

Firing a transition $t_j \in T$ is defined by the transformation marking M_aC into a new marking $M_{a+1}C$, when holds $FIRE(t_j)$ = TRUE, for each $i \in \langle 1,k \rangle$, $m \in \langle 1,h \rangle$:

$$M_{a+1}C((n,c)_{m,i}) = M_aC((n,c)_{m,i}) + QC((n,c)_{m,i,j}) - IC((n,c)_{m,i,j}) \quad (7)$$

Generelized Petri net is called a deterministic Petri net, if $UP=1 \wedge UT=1$ and non-deterministic Petri nets, if UP and UT are finite sets of probability, or values of membership function.

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