ON FUZZY CONVERGENCE OF FUZZY OBSERVABLES IN F-QUANTUM SPACES

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An F-quantum space as alternative axiomatic model for quantum mechanics has been suggested by B. Riečan in [1] .

An F-quantum space is couple (Ω, M), where Ω is a nonvoid set and $M \subset [0, 1]$ such that

(i) if $1(\omega) = 1$ for any $\omega \in \Omega$, then $1 \in M$,

(ii) $a \in M$ implies $1 - a = a^{\perp} \in M$,

(iii) if $1/2(\omega) = 1/2$ for any $\omega \in \Omega$, then $1/2 \notin M$,

(iv) $\bigcup_{n=1}^{\infty} a_n = \sup a_n \in M$ whenever $\{a_n\}_{n=1}^{\infty} \subset M$.

In the F-quantum space an F-observable is analogue of a random variable and an F-state is a generalization of a probability measure.

Let (Ω, M) be an F-quantum space. According to [2], we define a system

$K(M) = \{ A \subset \Omega : \exists a \in M$ such that $\{\omega \in \Omega : a(\omega) > 1/2\} \subset A \subset$
$\subset \{\omega \in \Omega : a(\omega) \geqq 1/2\}\}$.

The system $K(M)$ is a σ-algebra of subsets of Ω. If m is an F-state then the mapping $P_m : K(M) \longrightarrow [0, 1]$ defined via $P_m(A) = m(a)$, is a probability measure on $K(M)$.

In [3] A. Dvurečenskij proved the following proposition:

Let x be an F-observable on (Ω, M). Then there is a K(M)--measurable random variable $f : \Omega \longrightarrow R$ such that

(1) $\{\omega \in \Omega : x(E)(\omega) > 1/2\} \subset f^{-1}(E) \subset \{\omega \in \Omega : x(E)(\omega) \geq 1/2\}$
for any $E \in B(R)$.
Conversely, if f is a K(M)-measurable random variable, then there is an F-observable x with (1).

Let A, B $\in$ K(M). We say that the sets A, B are fuzzy equal with respect to M and we write $A \underset{F}{=} B$, iff there is a fuzzy set $c \in M$ such that $A \triangle B = A \cap B^c \cup A^c \cap B \subset \{\omega \in \Omega : c(\omega) = 1/2\}$, where $A^c = \Omega - A$.

Let f, g be K(M)-measurable random variables. We say that f, g are fuzzy equal with respect to M, $f \underset{F}{=} g$, iff there is a $c \in M$ such that $\{\omega \in \Omega : f(\omega) \neq g(\omega)\} \subset \{\omega \in \Omega : c(\omega) = 1/2\}$.

Let x, y be F-observables of (Ω, M). We say that x, y are fuzzy equal with respect to M, $x \underset{F}{=} y$, iff there is a $c \in M$ such that $\{\omega \in \Omega : x(E) \cap y(E)^{\perp}(\omega) > 1/2\} \cup \{\omega \in \Omega : x(E)^{\perp} \cap y(E)(\omega) > 1/2\} \subset$
$\subset \{\omega \in \Omega : c(\omega) = 1/2\}$.

The sum of two F-observables has been studied in [4].

We say that a seqence $\{x_n\}_{n=1}^{\infty}$ of F-observables (a sequence $f_n\,{}_{n=1}^{\infty}$ of K(M)-measurable functions) converges to an F-observable x (a K(M)-measurable function f)

(i) fuzzy everywhere with respect to M, iff for every $\varepsilon > 0$
$\bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} (x - x_n)([-\varepsilon, \varepsilon]) \underset{F}{=} 1$ $\left(\bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} (f-f_n)^{-1}([-\varepsilon, \varepsilon]) \underset{F}{=} \Omega\right)$;

(ii) fuzzy uniformly with respect to M, iff for every $\varepsilon > 0$ there is an integer k such that $(x - x_n)([-\varepsilon, \varepsilon]) \underset{F}{=} 1$ $((f - f_n)^{-1}([-\varepsilon, \varepsilon]) \underset{F}{=} \Omega)$, for all $n \geq k$;

(iii) almost everywhere in F-state m, iff for every $\varepsilon > 0$
$m(\bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} (x-x_n)([-\varepsilon, \varepsilon])) = 1$ $(P_m((f-f_n)^{-1}([-\varepsilon, \varepsilon])) = 1)$;

(iv) in an F-state m (a measure P_m), iff for every $\varepsilon > 0$

$\lim_{n\to\infty} m((x - x_n)([-\varepsilon,\varepsilon])) = 1$ $(\lim_{n\to\infty} P_m(f - f_n)^{-1}([-\varepsilon,\varepsilon])) = 1)$.

Analogously we may define the notions of a convergence uniform almost everywhere, fuzzy almost uniform, e.t.c.

THEOREM. Let x, x_n, $n \geq 1$, be F-observables of (Ω, M) and f, f_n, $n \geq 1$, be random variables satisfying (1). A sequence $\{x_n\}_{n=1}^{\infty}$ converges to x in an arbitrary sense from (i) through (iv) if and only if the sequence $\{f_n\}_{n=1}^{\infty}$ converges to f in the corresponding sense.

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