A Few Remarks on Dependence

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The fact that, analogously to random events, also the fuzzy objects and fuzzy quantities, can be more or less dependent is intuitively accepted and quite evident in many practical situations. However, its formal representation is still discutable and problematic. Generally, we can consider the situation where the degree in which a fuzzy set includes an element depends on the degree in which it includes another element, or the situation where it depends on the degree in which the same element belongs to another fuzzy set.

Formally, if A and B are fuzzy subsets of the same universum $\mathcal{U}$, and with the membership functions f_A and f_B, respectively, and if $x \in \mathcal{U}$, $y \in \mathcal{U}$ then we can be faced to the problem to express the dependence of the value $f_A(x)$ on $f_A(y)$ or the dependence of $f_A(x)$ on $f_B(x)$.

One of the principal differences between the probabilistic and fuzzy theoretical approaches to the uncertainty is the mutual polarity between local and global principles. Contrary to the global probabilistic approach via the measures of whole sets, the fuzzy theoretical concepts are based on the local properties, on the membership function values defined for each point and independent on the size or form of the whole fuzzy set. It means that the, theoretically possible and in literature already realized, endeavour to define "fuzzy measures" analogous to probabilistic measures is unadequate to the nature of the fuzziness. The "conditional measures" and "conditional fuzzy membership functions" which can be derived in this way cannot, by the author's opinion, satisfy the demands of the adequate description of the conditional fuzziness, as well.

The main purpose of this contribution is to suggest another approach to the conditionality of fuzzy phenomena. Let us consider the first one of the two problems formulated above. If A is a fuzzy set with the membership function f_A then we solve the problem how to define the dependence of the validity of $x \in A$ on the fact that $y \in A$ for other elements y.

Let us consider the Cartesian product $A \times A$ as a fuzzy subset of $\mathcal{U} \times \mathcal{U}$ with the membership function

$$f_{A\times A}(x,y) = \min\left(f_A(x), f_A(y)\right), \tag{1}$$

which value represents the possibility of the conjunction of $x \in A$ and $y \in A$. Then the "conditional possibility"

$$f_{A|A}\left(x|y \in A\right) \qquad \text{for} \quad y \in \mathcal{U},$$

which represents the possibility that $x \in A$ under the condition that $y \in A$ can be defined as the function over $\mathcal{U}$ such that

$$f_{A|A}(x|y) = f_{A\times A}(x,y)/f_{A\times A}(y,y). \tag{2}$$

Evidently $f_{A|A}(x|y)$ is a membership function defined over $\mathcal{U}$ and it represents the membership function of A under the condition that a fixed element of $y \in \mathcal{U}$ belongs also to A.

The procedure described above can be generalized to the case of two fuzzy subsets A, B of $\mathcal{U}$ with membership functions f_A and f_B. It is possible to define the product set $A \times B$ with $f_{A\times B}$ analogously to (1) and $f_{A|B}$ analogously to (2). Using this method we can describe the second type of dependence mentioned in the introductory paragraphs.

At the end of this contribution, it could be interesting to mention one interdisciplinary connection in spite of the fact that its relevance to the considered problem is very free. The classical membership function $f_A : \mathcal{U} \to \langle 0,1\rangle$ can be regarded as a one–parametric system of binary probabilities p_x such that the probability of the "random event" A is $p_x(A) = f_A(x)$ and the probability of its complement $\mathcal{U} - A$ is $p_x(\mathcal{U} - A) = 1 - f_A(x) = f_{\mathcal{U}-A}(x)$, for any value of the parameter $x \in \mathcal{U}$. The interdisciplinary connection (may be interesting, may be quite odd) consists in the fact that the one–parameter system of binary probabilities, understood also as the conditional probability

$$p_x(\cdot) = p(\cdot|x),$$

defines a binary communication channel known and investigated in the information theory. In this sense the membership function of the fuzzy set A is a noisy communication channel with the input and output alphabets

$\mathcal{U}$ and representing the noise (uncertainty, vagueness) connected with the recognition of the set $A \subset \mathcal{U}$. This view at the fuzzy sets as communication channel opens the possibility to use the probabilistic conditionality of channels for the investigation of some features of the conditionality between fuzzy sets and fuzzy phenomena.

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