FUZZY LOGIC LEARNING PROCESS

J.B. Kiszka*, M.M. Gupta**

*** Electrical Engineering**
Lakehead University
Thunder Bay, Ontario
P7B 5E1 Canada

**** College of Engineering**
University of Saskatchewan
Saskatoon, Sask.
S7N 0W0 Canada

Abstract: An inhibition process is treated by means of fuzzy logic. An unified theory of fuzzy logic neural systems is used to investigate a competitive learning process.

1. Introduction.

In [1], [2], [3], [4], and [5] we have outlined an unified theory of fuzzy logic neural systems. A major idea behind the theory is based on an assumption that a single neuron can be characterized by a set of fuzzy rules and fuzzy relational equations.

An inhibition process is a psychical activity imposing restrain upon another activity. We assume that the inhibition process is inherently vague.

We show how it is implemented with the help of the theory of fuzzy logic neural systems.

We use the fuzzy inhibition process to realize a competitive learning.

2. Inhibition Process.

Consider a fuzzy neural network shown in Figure 1. Each neuron receives an input from outside word and an output from opposite unit.

The more strongly any particular neuron responds to an incoming inputs, the more it inhibits the other unit. Let the inhibition process be characterized by the following set of fuzzy rules and fuzzy relational equations [1].

For neuron No. I

$$\left\{ \text{IF } X^{I}_{n(i)} \text{ and } U^{1}_{n(i)} \text{ and } Y^{2}_{n(i)} \text{ then } X^{I}_{n+1(i)} \text{ and } Y^{1}_{n(i)}, \text{ ALSO} \right\} \qquad (1)$$

where

$n = 0, 1, 2, 3, \ldots$; and $i = 1, 2, 3, \ldots, I$

and

$$\left.\begin{aligned} X^{I}_{n+1} &= X^{I}_{n} \circ R^{I}_{1} \,\Delta\, U^{1}_{n} \circ R^{I}_{2} \,\Delta\, Y^{2}_{n} \circ R^{I}_{3} && \text{(a)} \\ Y^{1}_{n} &= X^{I}_{n} \circ R^{I}_{4} \,\Delta\, U^{1}_{n} \circ R^{I}_{5} \,\Delta\, Y^{2}_{n} \circ R^{I}_{6} && \text{(b)} \end{aligned}\right\} \qquad (2)$$

where

$$R_1^I = \bigvee_i \left\{ X_{n(i)}^I \wedge X_{n+1(i)}^I \right\}, \ldots, R_6^I = \bigvee_i \left\{ Y_{n(i)}^2 \wedge Y_{n(i)}^1 \right\}$$

For neuron No. II

$$\left\{ \text{IF } X_{n(i)}^{II} \text{ and } U_{n(i)}^2 \text{ and } Y_{n(i)}^1 \text{ then } X_{n+1(i)}^{II} \text{ and } Y_{n(i)}^2, \text{ALSO} \right\} \tag{3}$$

and

$$\left.\begin{aligned} X_{n+1}^{II} &= X_n^{II} \circ R_1^{II} \,\Delta\, U_n^2 \circ R_2^{II} \,\Delta\, Y_n^1 \circ R_3^{II} && \text{(a)} \\ Y_n^2 &= X_n^{II} \circ R_4^{II} \,\Delta\, U_n^2 \circ R_5^{II} \,\Delta\, Y_n^1 \circ R_6^{II} && \text{(b)} \end{aligned}\right\} \tag{4}$$

In order to know how much does a sequence of Y_n^1 output inhibit a memory X_n^I of neuron No. I, it is necessary to combine Equations (2) and (3) and solve them with respect to a discrete-time variable n.

From Equations 2a), 2b), and 4b) we get

$$\begin{aligned} X_{n+1}^I = X_n^I \circ \left(R_1^I \,\Delta\, R_4^I \circ R_3^I \right) \Delta\, X_n^{II} \circ R_4^{II} \circ R_6^I \circ R_3^I \\ \Delta\, U_n^1 \circ \left(R_2^I \,\Delta\, R_5^I \circ R_3^I \right) \Delta\, U_n^2 \circ R_5^{II} \circ R_6^I \circ R_3^I \\ \Delta\, Y_n^1 \circ R_6^{II} \circ R_6^I \circ R_3^I \end{aligned} \tag{5}$$

Solving Equation (5) with respect to n we have

$$X_K^I = X_0^I \circ \left\{ R_1^I \Delta R_4^I \circ R_3^I \right\} \Delta \bigwedge_{i=1}^{k-1} X_i^{II} \circ \left\{ R_4^{II} \circ R_6^{II} \circ R_3^I \right\} \circ$$
$$\circ \left\{ R_1^I \Delta R_4^I \circ R_3 \right\}^{k-i-1} \Delta \bigwedge_{i=1}^{k-1} U_i^1 \circ \left\{ R_2^I \Delta R_5^I \circ R_3^I \right\} \circ$$
$$\circ \left\{ R_1^I \Delta R_4^I \circ R_3^I \right\}^{k-i-1} \Delta$$
$$\bigwedge_{i=1}^{k-1} U_i^2 \circ \left\{ R_5^{II} \circ R_6^I \circ R_3^I \right\} \circ \left\{ R_1^I \Delta R_4^I \circ R_3^I \right\}^{k-i-1}$$
$$\Delta \bigwedge_{i=1}^{k-1} Y_i^1 \circ \left\{ R_6^{II} \circ R_6^I \circ R_3^I \right\} \circ \left\{ R_1^I \Delta R_4^I \circ R_3^I \right\}^{k-i-1} \qquad (6)$$

From Equation (6) we see that the current experience X_K^I of neuron No. I is determined by the initial experience term X_0^I, the sequence of experience X_i^{II} of neuron No. II, the input signal terms U_i^2 and U_i^2, and finally is inhibited by the output signal term Y_i^1.

It is to say that $\bigwedge_{i=1}^{k-1} Y_i^1 \circ \left\{ R_6^{II} \circ R_6^I \circ R_3^I \right\} \circ \left\{ R_1^I \Delta R_4^I \circ R_3^I \right\}^{k-i-1}$ contributes to the inhibition of X_K^I.

3. Fuzzy Logic Competitive Learning Process.

The basic architecture of a fuzzy logic competitive learning process is illustrated in Figure 2. The fundamental building block of a competitive learning system is the inhibition process outline in Chapter 2.

The fuzzy logic competitive learning process consists of two layers of the fuzzy inhibition processes. Each layer has its own input signals. Each level connects with the neighboring level, and has inhibitory connections to neurons in its own level.

The units within onelayer compete with on another.

Neuron No. I competes with neuron No. III, and unit No. II is in rivalry with unit No. IV.

Elements No. I and No. II, as well units No. III and No. IV develop an alliance with mutually reinforcing connections. A neuron learns if it wins rivalry with other neurons in its layer. Mathematically, this learning process can be stated by the following set of fuzzy rules and fuzzy equations [1].

For unit No. I

$$\left\{ \text{IF } X^{I}_{n(i)} \text{ and } U^{1}_{n(i)} \text{ and } Y^{1}_{n(i)} \text{ and } W^{2}_{n(i)} \text{ then } X^{I}_{n+1(i)} \text{ and } W^{1}_{n(i)}\text{, ALSO} \right\} \quad (7)$$

and

$$\left.\begin{aligned} X^{I}_{n+1} &= X^{I}_{n} \circ R^{I}_{1} \,\Delta\, U^{1}_{n} \circ R^{I}_{2} \,\Delta\, Y^{1}_{n} \circ R^{I}_{3} \,\Delta\, W^{2}_{n} \circ R^{I}_{4} \quad &(a)\\ W^{1}_{n} &= X^{I}_{n} \circ R^{I}_{5} \,\Delta\, U^{1}_{n} \circ R^{I}_{6} \,\Delta\, Y^{1}_{n} \circ R^{I}_{7} \,\Delta\, W^{2}_{n} \circ R^{I}_{8} \quad &(b)\end{aligned}\right\} \quad (8)$$

For unit No. II

$$\left\{ \text{IF } X^{II}_{n(i)} \text{ and } V^{1}_{n(i)} \text{ and } W^{1}_{n(i)} \text{ and } Y^{2}_{n(i)} \text{ then } X^{II}_{n+1(i)} \text{ and } Y^{1}_{n(i)}\text{, ALSO} \right\} \quad (9)$$

and

$$\left.\begin{aligned} X^{II}_{n+1} &= X^{II}_{n} \circ R^{II}_{1} \,\Delta\, V^{1}_{n} \circ R^{II}_{2} \,\Delta\, W^{1}_{n} \circ R^{II}_{3} \,\Delta\, Y^{2}_{n} \circ R^{II}_{4} \quad &(a)\\ Y^{1}_{n} &= X^{II}_{n} \circ R^{II}_{5} \,\Delta\, V^{1}_{n} \circ R^{II}_{6} \,\Delta\, W^{1}_{n} \circ R^{II}_{7} \,\Delta\, Y^{2}_{n} \circ R^{II}_{8} \quad &(b)\end{aligned}\right\} \quad (10)$$

For neuron No. III

$$\left\{ \text{IF } X^{III}_{n(i)} \text{ and } U^{2}_{n(i)} \text{ and } Y^{2}_{n(i)} \text{ and } W^{1}_{n(i)} \text{ then } X^{III}_{n+1(i)} \text{ and } W^{2}_{n(i)}\text{, ALSO} \right\} \quad (11)$$

and

$$\left.\begin{aligned} X^{III}_{n+1} &= X^{III}_{n} \circ R^{III}_{1} \,\Delta\, U^{2}_{n} \circ R^{III}_{2} \,\Delta\, Y^{2}_{n} \circ R^{III}_{3} \,\Delta\, W^{1}_{n} \circ R^{III}_{4} \quad &(a)\\ W^{2}_{n} &= X^{III}_{n} \circ R^{III}_{5} \,\Delta\, U^{2}_{n} \circ R^{III}_{6} \,\Delta\, Y^{2}_{n} \circ R^{III}_{7} \,\Delta\, W^{1}_{n} \circ R^{III}_{8} \quad &(b)\end{aligned}\right\} \quad (12)$$

For neuron No. IV

$$\left\{ \text{IF } X^{IV}_{n(i)} \text{ and } V^{2}_{n(i)} \text{ and } W^{2}_{n(i)} \text{ and } Y^{1}_{n(i)} \text{ then } X^{IV}_{n+1(i)} \text{ and } Y^{2}_{n(i)}, \text{ ALSO} \right\} \tag{13}$$

and

$$\left.\begin{aligned} X^{IV}_{n+1} &= X^{IV}_{n} \circ R^{IV}_{1} \,\Delta\, V^{2}_{n} \circ R^{IV}_{2} \,\Delta\, W^{2}_{n} \circ R^{IV}_{3} \,\Delta\, Y^{1}_{n} \circ R^{IV}_{4} && \text{(a)} \\ Y^{2}_{n} &= X^{IV}_{n} \circ R^{IV}_{5} \,\Delta\, V^{2}_{n} \circ R^{IV}_{6} \,\Delta\, W^{2}_{n} \circ R^{IV}_{7} \,\Delta\, Y^{1}_{n} \circ R^{IV}_{8} && \text{(b)} \end{aligned}\right\} \tag{14}$$

It is more convenient to display the fuzzy logic competitive learning process in terms of vector-matrix notation.

$$\begin{bmatrix} X^{I}_{n+1} \\ X^{II}_{n+1} \\ X^{III}_{n+1} \\ X^{IV}_{n+1} \end{bmatrix} = \begin{bmatrix} X^{I}_{n} & U^{1}_{n} & Y^{1}_{n} & W^{2}_{n} \\ X^{II}_{n} & V^{1}_{n} & W^{1}_{n} & Y^{2}_{n} \\ X^{III}_{n} & U^{2}_{n} & Y^{2}_{n} & W^{1}_{n} \\ X^{IV}_{n} & V^{2}_{n} & W^{2}_{n} & Y^{1}_{n} \end{bmatrix} * \begin{bmatrix} R^{I}_{1} & R^{II}_{1} & R^{III}_{1} & R^{IV}_{1} \\ R^{I}_{2} & R^{II}_{2} & R^{III}_{2} & R^{IV}_{2} \\ R^{I}_{3} & R^{II}_{3} & R^{III}_{3} & R^{IV}_{3} \\ R^{I}_{4} & R^{II}_{4} & R^{III}_{4} & R^{IV}_{4} \end{bmatrix} \tag{15}$$

where * means (o, Δ)-composition

and

$$\begin{bmatrix} W^{1}_{n} \\ Y^{1}_{n} \\ W^{2}_{n} \\ Y^{2}_{n} \end{bmatrix} = \begin{bmatrix} X^{I}_{n} & U^{1}_{n} & Y^{1}_{n} & W^{2}_{n} \\ X^{II}_{n} & V^{1}_{n} & W^{1}_{n} & Y^{2}_{n} \\ X^{III}_{n} & U^{2}_{n} & Y^{2}_{n} & W^{1}_{n} \\ X^{IV}_{n} & V^{2}_{n} & W^{2}_{n} & Y^{1}_{n} \end{bmatrix} * \begin{bmatrix} R^{I}_{5} & R^{II}_{5} & R^{III}_{5} & R^{IV}_{5} \\ R^{I}_{6} & R^{II}_{6} & R^{III}_{6} & R^{IV}_{6} \\ R^{I}_{7} & R^{II}_{7} & R^{III}_{7} & R^{IV}_{7} \\ R^{I}_{8} & R^{II}_{8} & R^{III}_{8} & R^{IV}_{8} \end{bmatrix} \tag{18}$$

Equation (15) determines the vector of intended experience of the competitive learning process in terms of fuzzy matrices $R^{I}_{1}, \ldots, R^{IV}_{4}$.

Equation (16) furnishes the vector of outputs of the process in terms of the past experience and the inputs.

It is worth to stress that Equations (15) and (16) together with the fuzzy logic neural processor [4] may constitute very powerful technique to design and analyze the fuzzy logic competitive learning process.

4. Summary.

The inhibition process has been modeled in terms of fuzzy logic.Unified fuzzy equations of competitive learning process have been proposed. Techniques outlined here may be used to design VLSI circuitry of competitive learning in vague environment.

References.

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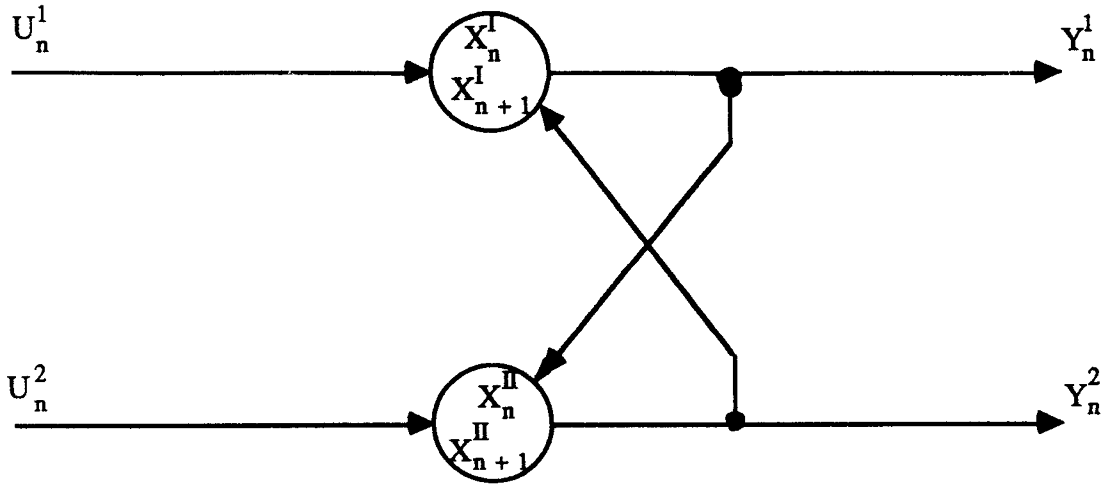

Figure 1. Fuzzy logic inhibition process

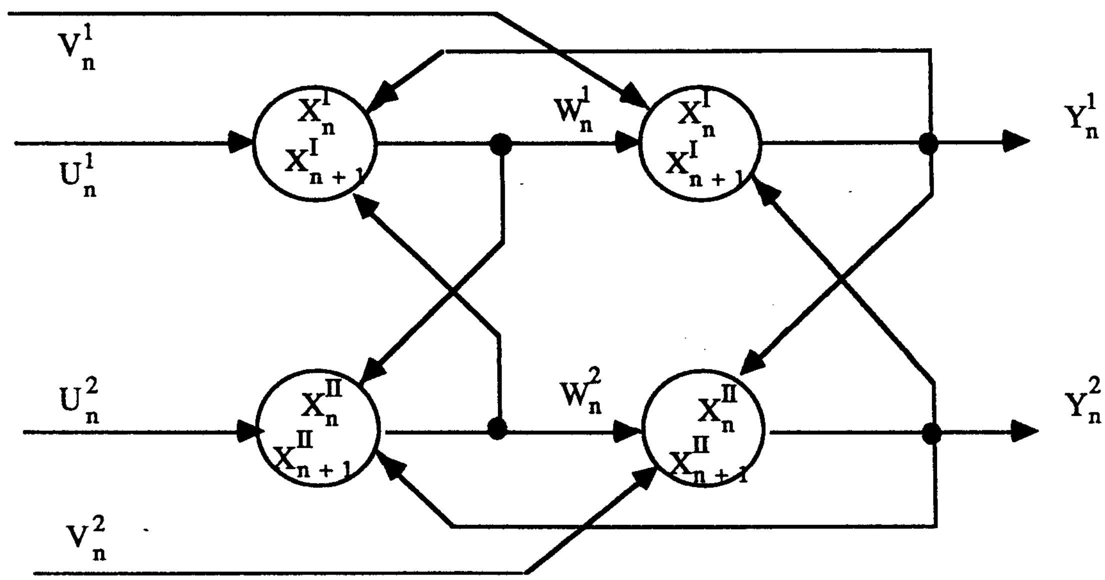

Figure 2. Fuzzy logic competitive learning process