

LOOKAHEAD TECHNIQUE FOR A FUZZY INFERENCE MECHANISM

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IN A MULTIPLE EXPERT SYSTEMS ENVIRONMENT

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Abstract

A practical and effective lookahead technique dedicated to a fuzzy inference mechanism is reported. The fuzzy solver is supposed to act within a multiple expert environment, supporting the competitive modelling and processing of non-crisp knowledge, by several cognitive entities (agents). The aim of the technique is to provide the most promising path that the fuzzy solver should follow during the resolute process. The selection is operated among the alternative strategies supplied by a group of agents involved in the achievement of the same goal.

The selection procedure is based on the principles of multicriterial decision making models. Starting from a fuzzy representation for such a model, a global fuzzy preference relation is constructed, which induces a crisp dominance relation between alternatives.

The best agent(set of rules) and its most recommendable strategy(rule), respectively, are determined, as elements of the corresponding sets of non-dominated alternatives.

The fuzzy inference mechanism will further operate only upon the selected alternatives.

Key words: multiple expert systems, fuzzy modelling,  
fuzzy preference relation, linguistic variable,  
fuzzy multicriterial decision making

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## 1. INTRODUCTION

The necessity to develop large, yet efficient and reliable solving environments yielded an ever increasing interest rate in conceiving rather sophisticated strategies of distributing knowledge and inferential power in several self-contained carriers that are able to act independently as well as to negotiate and aggregate their opinions and preferences in order to support the the achievement of common goals [1,4,6,7].

The difficulty of the task increases when non-crisp knowledge is involved [5].

The requests (problems to be solved), along with the input information supplied, such as formulated by the user, become a matter of individual interpretation for an agent [6,7].

The model developed and implemented by the authors, presented further in sections 3 - 6, considers this interpretative phase to be splitted into three parts:

- "tuning" of the information received, against that one residing by the agent (using the available fuzzy modelling relations and fuzzy modelling mappings);
- suitability estimation (a measure is produced concerning the agent's capacity to accomplish the demanded task);
- availability estimation (a measure is produced concerning the supporting circumstances favorable to the agent).

The request will finally be responded by that agent that proves to be both suitable and available to a highest degree.

This degree is estimated by the evaluation of specific criteria, such as certainty, possibility and efficiency of a given strategy in achieving the request.

The model supplied by the authors offers rather intuitive definitions for all the criteria involved in the selection models.

Both decisions, concerning the choice of an agent to act and of an alternative strategy to follow, are performed according to the principles of a concordant fuzzy representation for the multicriterial decision making problem [2,5,8].

Section 2 contains the basic theoretical support for the model. The implementation is made under GC-Lisp on IBM-PC/AT.

## 2. FUZZY MULTICRITERIA DECISION MAKING MODEL

We shall further consider a suitable fuzzy model for the multicriteria decision making problem.

Thus, according to [8], a decision situation is represented as a pair  $\langle X, P \rangle$ , where  $X$  denotes a set of competitive alternatives and  $P$  denotes a vector fuzzy preference relation  $P = \{ P_1, \dots, P_m \}$ , where each  $P_j$  represents a different preference relation, given by a membership function  $\mu_j : X \times X \rightarrow [0,1]$ .

For each membership function  $\mu_j$  there corresponds the crisp relation  $F_j$ :

$$F_j = \{ (x,y) \mid \Delta_j(x,y) = \mu_j(x,y) - \mu_j(y,x) \geq 0 \}$$

As well, the Pareto domination relation  $F_P = \bigcap_{j=1,m} F_j$  and the Pareto set  $X_P^{UND} = X_{\Pi}(F_P)$  of nondominated alternatives are introduced.

For one single preference relation, with membership function  $\mu$ , the unfuzzy set of nondominated alternatives is given by:

$$X^{UND}(\mu) = \{ x \mid \mu^{ND}(x) = 1 \}$$

where

$$\mu^{ND}(x) = 1 - \max_{y \in X} \mu^S(y,x)$$

and

$$\mu^S(x,y) = \begin{cases} \Delta(x,y) = \mu(x,y) - \mu(y,x), & \text{if } \Delta(x,y) \geq 0 \\ 0, & \text{if } \Delta(x,y) < 0 \end{cases}$$

As well,  $X^{UND}(\mu) = X_{\Pi}(F)$ , where  $F = \{ (x,y) \mid \Delta(x,y) \geq 0 \}$

In the case of a collection  $P$  of fuzzy preferences the elements of  $X_P^{UND}$  are obtained by using the so-called convolution of  $P$ . A convolution of fuzzy preferences is defined by a global fuzzy preference relation  $M = [X \times X, \mu(x,y)]$  with nonempty  $X^{UND}(\mu)$ . The membership function  $\mu$  is given by:

$$\mu(x,y) = f [ \mu_1(x,y), \dots, \mu_m(x,y) ]$$

A choice procedure is called effective when it produces an alternative (or some equivalent alternatives) from  $X_P^{UND}$ .

The convolution defined by the formula:

$$\mu_L(x, y) = \sum_{j=1, m} \lambda_j \times \mu_j(x, y)$$

where  $\lambda = \{ \lambda_1, \dots, \lambda_m \}$ ,  $\lambda \in \Lambda$  and  $\Lambda = \{ \lambda \mid \lambda_j \geq 0; \sum_{j=1, m} \lambda_j = 1 \}$  is effective [8].

The crisp relation associated to  $\mu_L$  is represented by:

$$F(\mu_L; \lambda) = \{ (x, y) \mid \sum_{j=1, m} \lambda_j \times \Delta_j(x, y) \geq 0 \}$$

with  $\lambda_j$  as above. The effectiveness of the convolution  $\mu_L$  is based on the equality :

$$X_{\Pi} [F(\mu_L; \lambda)] = X^{UND}(\mu_L; \lambda)$$

and the inclusion:

$$X^{UND}(\mu_L, \lambda) \subseteq X_P^{UND}, \text{ for all } \lambda \in \Lambda.$$

Fuzzy representations are preferable for multicriteria decision problems, since usually the initial Pareto set  $X_{\Pi}^K$  in crisp representations is too large and hard to check for the decision maker. The number of alternatives can be reduced and hence made more visible for the decision maker, by means of fuzziness. The only condition is to keep concordance between the two representations, namely to keep the order given by the dominance relation. According to [8], the two representations remain concordant if fuzziness is introduced in the following way:

$$\mu_j(x, y) = [K_j(x) - K_j(y)] / 2 \times d_j + 1/2$$

where  $d_j = \max_{x, y \in X} [K_j(x) - K_j(y)]$ , represents the scale range for criterion  $K_j$ . In [8] it is shown that  $X_j^* = X^{UND}(\mu_j)$  holds

where  $X_j^* = \{ x \mid \max_x K_j(x) \}$ .

Thus, the specified fuzzy representation is concordant to the initial crisp representation.

### 3. AGENTS, RULES AND FACTS

We consider an agent  $A$  to be a member of a set  $\mathcal{A}$ , where each element is represented by a collection of rules  $R^A$ , dedicated to the achievement of some goal  $G^A$ .

The rules in  $R^A$  signify alternative modalities to attain  $G^A$ . The goal  $G^A$  is treated as the "skill" provided for agent A.

As we shall see further, some agents may be more skillful than others, depending on the various circumstances they are facing.

The collection  $R^A = \{ R_1^A, \dots, R_r^A \}$  consists of several backward-type rules  $R_i^A$  of the form:

(conclusion  $\leftarrow$  ((premise-1)...(premise-n))

We consider that rules are expressing the dependency of a single parameter value, specified within the conclusion, on the value of several other parameters, specified by the premises.

The values of parameters can be either crisp or linguistic. Thus, a parameter can also be seen as a linguistic variable  $L$ , with the set of linguistic values  $\{ L_1, \dots, L_n \}$ , defining the corresponding fuzzy sets on the universe

$X = \{x_1, \dots, x_s\}$ .

Every premise is supposed to have three different weighted features attached to it:

- expectation (  $ex(p)$  ): expresses how much the user counts on that premise to be fulfilled (  $0 \leq ex \leq 1$  );
- fuzziness (  $fz(p)$  ): expresses the degree of fuzziness of the assumption stated by the premise, such as resulted from the "tuning" with the currently available information;
- usefulness (  $uf(p)$  ): expresses how important that premise is considered to be, from the point of view of the user, for the overall accomplishment of the conclusion (  $0 < uf \leq 1$  ).

Finally, we let the availability of a premise to be the product:

$$av(p) = ex(p) \times fz(p)$$

expressing the overall degree of "reachability" attached to it.

As a sequel, the features of the premises  $p$  determine consequent features of the rule  $R_i^A$  as a whole, concerning its power to attain  $G^A$ .

We shall define just three such features, since we consider them overwhelming. Of course, many others can still be imagined. Thus, we shall consider:

- the possibility degree of  $R_i^A$  fulfilling  $G^A$ , defined as:

$$\text{pos} ( R_i^A ) = \sum_P \text{av} ( p ) , p \in P_i^A$$

- the certainty degree of  $R_i^A$  fulfilling  $G^A$ , defined as:

$$\text{cer} ( R_i^A ) = \min_P \text{av} ( p ) , p \in P_i^A$$

- the efficiency of  $R_i^A$  in fulfilling  $G^A$ , defined as:

$$\text{eff} ( R_i^A ) = [ \sum_P \text{av} ( p ) \times \text{uf} ( p ) ] \times \text{str} ( R_i^A ) , p \in P_i^A$$

where  $P_i^A$  is the set of premises in  $R_i^A$  and  $\text{str} ( R_i^A )$  represents the strength of the connection between premises and conclusion in rule  $R_i^A$  (to be defined in section 4).

#### 4. TUNING ISSUES

During the solving process, the provenience of values for the features of the premises results from three sources:

- values due to the facts of the knowledge base;
- "on-site" input information, elicited from the user;
- ascendent transfer of values during the upward traversal of the goal decomposition tree.

Thus, the value of the fuzziness feature, included by the premise description is supposed to reflect the very "tuning" degree between the premise and the currently available knowledge.

We propose a definition for the fuzziness feature seen as a conditional function, illustrating the "tuning" degree between two concepts, which to include the case of crisp values:

$$\text{fz} ( Y | X ) = \begin{cases} \mu_Y ( X ) , & \text{when } X \text{ is some crisp value of a universe } U \\ & \text{and } Y \text{ is some fuzzy set defined on } U \\ \max_{x \in U} \min ( \mu_X ( x ) , \mu_Y ( x ) ) , & \text{when both } X \text{ and } Y \\ & \text{are fuzzy sets on the same universe } U \end{cases}$$

Under these circumstances, we can also define  $\text{str} ( R_i^A )$ , for instance by:

$$\text{str} ( R_i^A ) = \min ( 1 , 1 + \text{fz} ( G^A | LG_e^A ) - \text{ffz} ( P_i^A ) )$$

where  $\text{ffz} ( P_i^A ) = \prod_{j=1,q} \text{fz} ( P_j | L_e^j ) , P_j \in P_i^A$ .

$LG_e^A$  and  $L_e^j$  are supposed to represent the most

highly expected alternatives for the evolution of the parameters referred by the goal  $G^A$  and premise  $p_j$  respectively.

And of course, a value for the global strength of the whole collection  $R^A$  to fulfill the goal  $G^A$ , can be estimated by:

$$\text{str}(R^A) = \min_i \text{str}(R_i^A)$$

## 5. BEST AGENTS...

We shall further consider two criteria to be used in differentiating agents.

The first one, that we shall call suitability is intended to express the "competence" of the agent in achieving the goal  $G^A$ .

We define it by means of the most promising rule, namely the rule with the highest efficiency degree attained:

$$\text{suit}(A) = \max_{R \in R^A} \text{eff}(R)$$

The second one, that will be called availability, is meant to show how "performant" the agent  $A$  can afford to be in fulfilling  $G^A$ , taking into account the supporting circumstances.

It is defined in terms of the rule with the most promising premises:

$$\text{ava}(A) = \max_{R \in R^A} \text{pos}(R)$$

Starting from the suitability and availability features we can define a fuzzy two-criteria decision-making model, which offers one modality to choose the most adequate agents for the task of fulfilling  $G^A$ . The set of competitive alternatives is the very set  $\mathcal{A}$  of agents.

The vector fuzzy preference relation will be taken as the collection of the suitability (R-suit) and availability (R-ava) relations, both treated as fuzzy preference relations on:

$$P = \{ P_1 = \text{R-suit}, P_2 = \text{R-ava} \}$$

where,

$$\text{R-suit}(A_1, A_2) = \frac{\text{suit}(A_1) - \text{suit}(A_2)}{2 \times d\text{-suit}} + 1/2$$

with  $d\text{-suit} = \max_{A_1, A_2 \in \mathcal{A}} [\text{suit}(A_1) - \text{suit}(A_2)]$

and, analogously,

$$R\text{-ava}(A_1, A_2) = \frac{\text{ava}(A_1) - \text{ava}(A_2)}{2 \times d\text{-ava}} + 1/2$$

with  $d\text{-ava} = \max_{A_1, A_2 \in \mathcal{A}} [\text{ava}(A_1) - \text{ava}(A_2)]$ .

We consider that both criteria are equally meaningful for discriminating between two agents. Hence, an importance degree of 1/2 will be associated to each of them and the following convolution is built, which is known to be effective [8]:

$$\mu_{ag}(A_1, A_2) = 1/2 \times R\text{-suit}(A_1, A_2) + 1/2 \times R\text{-ava}(A_1, A_2)$$

The global fuzzy preference relation P induces a crisp relation  $D_P$  :

$$D_P = \{ (A_i, A_j) \mid \mu_{ag}(A_i, A_j) > 0 \}$$

where

$$\mu_{ag}^S(A_i, A_j) = \begin{cases} \Delta_P(A_i, A_j) = 1/2 \Delta_{R\text{-suit}}(A_i, A_j) + 1/2 \Delta_{R\text{-ava}}(A_i, A_j) & \text{if } \Delta_P(A_i, A_j) > 0 \\ 0 & \text{if } \Delta_P(A_i, A_j) \leq 0 \end{cases}$$

and

$$\Delta_{R\text{-suit}}(A_i, A_j) = R\text{-suit}(A_i, A_j) - R\text{-suit}(A_j, A_i)$$

$$\Delta_{R\text{-ava}}(A_i, A_j) = R\text{-ava}(A_i, A_j) - R\text{-ava}(A_j, A_i)$$

Consequently, the best agents for achieving  $G^A$  are going to be determined, as elements of the unfuzzy set  $X^{\text{UND}}(\mu_{ag})$  of nondominated alternatives, according to  $D_P$  :

$$X^{\text{UND}}(\mu_{ag}) = \{ A \mid \mu_{ag}^{\text{ND}}(A) = 1 \}$$

where

$$\mu_{ag}^{\text{ND}}(A) = 1 - \max_{A_0 \in \mathcal{A}} \mu_{ag}^S(A_0, A)$$

Hence,

$$X^{\text{UND}}(\mu_{ag}) = \{ A \mid \max_{A_0 \in \mathcal{A}} \mu_{ag}^S(A_0, A) = 0 \}$$

that is,

$$X^{\text{UND}}(\mu_{ag}) = \{ A \mid \max_{A_0 \in \mathcal{A}} [\mu_{ag}^S(A_0, A) - \mu_{ag}^S(A, A_0)] = 0 \}$$



## 6. ...AND BEST RULES

In a quite similar manner, the most adequate rule to be used from a collection of rules can be determined, in case the most suitable agent (collection of rules) was already established for the task at hand.

Three criteria, namely certainty, possibility and efficiency are used to define the fuzzy preference relations  $P_j: R^A \times R^A \rightarrow [0,1]$ , contained in the vector fuzzy preference relation:

$$P = \{ P_1 = R\text{-cer}, P_2 = R\text{-pos}, P_3 = R\text{-eff} \}$$

The preference relations have the following definitions:

$$R\text{-cer}(R_1, R_2) = [\text{cer}(R_1) - \text{cer}(R_2)] / 2 \times d\text{-cer} + 1/2$$

$$\text{with } d\text{-cer} = \max_{R_1, R_2 \in R^A} [\text{cer}(R_1) - \text{cer}(R_2)],$$

$$R\text{-pos}(R_1, R_2) = [\text{pos}(R_1) - \text{pos}(R_2)] / 2 \times d\text{-pos} + 1/2$$

$$\text{with } d\text{-pos} = \max_{R_1, R_2 \in R^A} [\text{pos}(R_1) - \text{pos}(R_2)],$$

$$\text{and } R\text{-eff}(R_1, R_2) = [\text{eff}(R_1) - \text{eff}(R_2)] / 2 \times d\text{-eff} + 1/2$$

$$\text{with } d\text{-eff} = \max_{R_1, R_2 \in R^A} [\text{eff}(R_1) - \text{eff}(R_2)].$$

In order to reflect the contribution of each criterion to the overall comparison of two rules, we may take for instance

$$\lambda_1 = 1/4, \lambda_2 = 1/4, \lambda_3 = 1/2.$$

Hence, the convolution of the three criteria is defined by:

$$\mu_r(R_1, R_2) = 1/4 R\text{-cer}(R_1, R_2) + 1/4 R\text{-pos}(R_1, R_2) + 1/2 R\text{-eff}(R_1, R_2)$$

The effectiveness is certain and thus, the most recommendable rules are going to be determined, as elements of the unfuzzy set of nondominated alternatives, according to the crisp dominance relation induced by  $P$ :

$$X^{\text{UND}}(\mu_r) = \{ R \in R^A \mid \max_{R_0 \in R^A} [\mu_r(R_0, R) - \mu_r(R, R_0)] = 0 \}$$

## 7. Conclusion

A practical solution was presented for the determination of the best agents and of the most recommendable rules to exploit for the achievement of a specified goal, within a multiple agent environment, based on the competitive processing of non-crisp knowledge.

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