General Fuzzy Number's Operation and Property

Lei Guoming Yue Changan
Handan PrefctuRe Education College Handan Hebei China

I. The spread of Rational Grey Number

In message 〔I〕 We have given out and spread the abstract definitiaus of interval-type grey number (or administrative-type grey number) and information--type grey number (or Deng grey number). On this base We will spread rational grey number as follows:

Definiton 1: supposing A is a complex interval grey number set, B is a complex Deng grey number set, thus we call $c=A\cup B$ a complex rational grey number set, the elements in it are called complex rational grey numbers, and we write them as Ga, b.

Obviously when a complex rational grey number set is an interval grey number set, it is a fuzzy set, so comlex rational grey number is also called general fuzzy numher.

Definition 2. ①. When $a\leqslant b$, complx rational grey number $G_{a,b}$ is called finite complex rational grey number.

②. When $a>b$, complex rational grey number. $G_{a.b}$ is called infinite complex rational grey number.

Directly infinite complex rational grey number $G_{a,b}$ is the part of real number set apart from $I_{b,a}$. It is an interval that gets round countless big points.

When $a=b$, $G_{a,b}=G_{a,a}=$ 〔a, a〕, thus grey number 〔a, a〕 $\forall x\in R$ can be written as $G_{x,x}=$ 〔x, x〕.
When $x=0$, $G_{0,0}=$ 〔0, 0〕.

Definition 3: When $a\neq 0$, $b\neq 0$, complex rational grey number $G_{a,b}$ and G are called one another's reversal complex rational grey number One is called the other's reversal complex rational grey number, and that of $G_{a,b}$ is written as G_{a} , b.

Definition 4: complex rational grey number $G_{a,b}$ and $G_{-b,-a}$ are called one another's opposite complex rational grey number. One is called the other's opposite comptex rational grey number, and that of $G_{a,b}$ is written as $-G_{a,b}$.

Definition 5: complex rational grey number $G_{o,o}$ is called zero complex rational grey number.

Ⅱ. The Operation of Complex Rational Grey Number

On the bases of the definitions of complex rational grey number, we research for their operations now Let's set: When $x=0$, $\frac{1}{x}=\infty$

Defivition 6:

①. Operation of addition($\oplus$) :

$G_{a,} \oplus G_{c,d} = \{ x+y \mid x \in G_{a,b},\ y \in G_{c,d} \}$

②. Operation of subtruction($\ominus$) :

$G_{a,b} \ominus G_{c,d} = \{ x-y \mid x \in G_{a,b},\ y \in G_{c,d} \}$

③. Operation of multiplication($\odot$) :

$G_{a,b} \odot G_{c,d} = \{ x \cdot y \mid x \in G_{a,b},\ y \in G_{c,d} \}$

④. Operation of division ($\oslash$) :

$G_{a,b} \oslash G_{c,d} = \{ \frac{x}{y} \mid x \in G_{a,b},\ y \in G_{c,d} \}$

$G_{c,d} \neq G_{o,o}$.

In the following paragraph we will study the opertion rules according to the operation definitions:

①. Addition

i The adding of two finite complex rational grey numbers is still a finite complex rational grey number:

that means:

$I_{a,b} \oplus I_{c,d} = I_{a+c,\ b+d}$.

$$Ia\underset{\sim}{.}b \oplus Ic.d = Ia+c\underset{\sim}{.}b+d.$$

$$Ia.b \oplus Ic\underset{\sim}{,}d = Ia+c\underset{\sim}{,}b+d. \qquad a \leqslant b, \quad c \leqslant d$$

ii The adding of a finite complex rational grey number and an infinite complex rational grey number is an infinite one.

That *means:*

$$Ia.b \oplus Ic.d = Ia+c.\ b+d.$$

$$Ia\underset{\sim}{.}b \oplus Ic.d = Ia+c\underset{\sim}{,}b+d = Ia,b\ Ic,\ d$$

$$Ia.\ b \oplus Ic\underset{\sim}{,}d = Ia+c\underset{\sim}{,}b+d \qquad a \leqslant b\ \ c > d$$

When $a+c \leqslant b+d$, *the adding of them are all real numbers*

iii The adding of two infinite complex rational grey numbers is the whole number axis. That means:

$$Ga.b \oplus Gc.d = (-\infty,\ +\infty)$$

$(\ a > b,\ \ c > d\ )$ *Here we omit the demonstration.*

②. *Subtruction*

i The difference of two infinite complex rational grey numbers is the whole number axis.

ii The difference of two finite complex rational grey numbwrs is a finite one.

That *means:*

$$Ia.b \ominus Ic.d = Ia-d.\ b-c$$

$$Ia\underset{\sim}{,}b \ominus Ic.d = Ia-d\underset{\sim}{.}b-c \qquad a \leqslant b$$

$$Ia.b \ominus Ic\underset{\sim}{,}d = Ia-d\underset{\sim}{,}b-c \qquad c \leqslant b$$

$$Ia\underset{\sim}{.}b \ominus Ic\underset{\sim}{,}d = Ia-d\underset{\sim}{,}b-c$$

iii The difference of an infinite Complex rational grey number and a finite one is an infinite one. When at least one of them is a complex Deng grey number, the differnce is a complex Deng grey number. That means:

$$Ga.b \ominus Gc.d = Ga-d.\ b-c$$

$$(\ a > b.\ \ c \leqslant d\ )$$

In a word, except two infinite complex rational grey numbers, the difference of any two complex rational grey numbers equals the sum of the minuend complex ratioonal gray number and the opposite number of the subtrahend complex rational grey number.

Here we omit the demonstration.

③. Multiplication

i zero complex rational grey number times any complex ratlonal grey number, the result is a zero complex rational gray number.

ii a non-zero cimplex rational grey number times an infinite complex rational grey number, the result is an infinite complex rational grey number.

When the upper and lower sign of finite complex rational grey numbers are different, the result of this multiplication is all real numbers.

When the upper and lower sign of finite complex rational grey numbers are both negative,

$Ga, b \odot Gc, d = Ge, f \qquad a \leqslant b < 0 \qquad a < d$

$e = min\{ad, bd\} \quad f = max\{ac, bc\}$

When the upper and lower sign of finite complex rational grey numbers are both positive,

$Ga, b \odot Gc, d = Ie, f \qquad o < a \leqslant b$

$c > d$

$e = min\{ac, bc\} \qquad f = max\{ad, bd\}$

iii the multiplication result of two infinite complex rational grey numbers is an infinite complex rational grey number.

That is:

$Ga, b \odot Gc, d = Ge, f \qquad a > b \qquad c > d$

$e = min\{ac, bd\} \quad f = max\{ad, bc\}$

iv The multiplilation result of two finite complex rational grey numbers is a finite one.

That is:

$G_{a,b} \odot G_{c,d} = G_{e,f} \quad a \leqslant b \quad c \leqslant d$

$e = \min\{ac, ad, bc, bd\}$

$f = \max\{ac, ad, bc, bd\}$

This demonstration is the same as that of interval grey number, so here We omit it.

④ Division: According to definition④,

We know as follows:

$$G_{a,b} \oslash G_{c,d} = \left\{ \frac{x}{y} \,\middle|\, \begin{matrix} x \in G_{a,b} \\ y \in G_{c,d} \end{matrix} \right\}$$

$$G_{c,d} \neq G_{o,o}.$$

$$G_{a,b} \odot G_{c,d}^{-1} = G_{a,b} \odot G_{\frac{1}{d},\frac{1}{c}} = \left\{ X \cdot \frac{1}{y} \,\middle|\, \begin{matrix} x \in G_{a,b} \\ y \in G_{c,d} \end{matrix} \right\}$$

$$\therefore G_{a,b} \oslash G_{c,d} = G_{a,b} \odot G_{c,d}^{-1}$$

That means, one complex rational grey number divided by another non-zero complex rational grey number equals this complex rational grey number times the other non-zero comptex rational grey number's reversal complex rational grey number.

From the above mentioned, we know that the four fundmental operations of arithmetic of rational grey numbers can all be kept. So the four fundmental operation of arithmetic of complex rational grey numbers are the spread of those rational ones.

Ⅱ、The properties of complex Rational Grey Numbers

Theorem 1. $G_{a,b} \oplus G_{o,o} = G_{a,b}$

$G_{a,b} \odot G_{o,o} = G_{o,o}.$

(Here we omit the demonstration)

Theorem 2. $\forall a \in R$

① $a \odot G_{c,d} = G_{ac,ad} \quad a \geqslant 0$

② $a \odot G_{c,d} = G_{ad,ac} \quad a < 0$

(Here we omit the demonstration)

Theorem 3. $\forall a \in R$

$a \oplus G_{c,d} = G_{a+c,\ a+b}$

Theorem 4. satisfies the exchange rules of addition and multiplication.

$G_{a,b} \oplus G_{c,d} = G_{c,d} \odot G_{a,b}$

$G_{a,b} \odot G_{c,d} = G_{c,d} \odot G_{a,b}$

(Here we omit the demonstration)

Theorem 5 satisfies the united rules of addition and multiplication.

① $G_{a,b} \oplus (G_{c,d} \oplus G_{e.f}) = (G_{a,b} \oplus G_{c,d}) \oplus G_{e,f}$

② $G_{a,b} \odot (G_{c,d} \odot G_{e.f}) = (G_{a,b} \odot G_{c,d}) \odot G_{e,f}$

proof: ①

i $G_{a.b}, G_{c,d}, G_{c,f}$ are all finite complex rational grey numbers. According to the rules of addition's operation, we know:

$G_{a,b} \oplus (G_{c,d} \oplus G_{e,f}) = G_{a,b} \oplus G_{e+e,\ d+f}$

$= G_{a+c+e,\ b+d+f} = G_{a+c,\ d+b} \odot G_{e,f}$

$= (G_{a,b} \odot G_{c,d}) \odot G_{e,f}$

ii Only one of $G_{a,b}$, $G_{c,d}$, $G_{e,f}$ is a finite complex rational grey number, let's suppose $G_{a,b}$ is a finite complex rational grey number.

$G_{a,b} \oplus (G_{c,d} \oplus G_{e,f}) = G_{a,b} \oplus (-\infty, +\infty) = (-\infty, +\infty)$

$(G_{a,b} \oplus G_{c,b}) \oplus G_{e,f} = G_{a+c,\ b+d}$ (infinite complex rational gray number) $+ G_{e,f} = (-\infty, +\infty)$

$\therefore G_{a,b} \oplus (G_{c,d} \oplus G_{e,f}) = G_{a,b} \oplus G_{c,d}) \oplus G_{e,f}$

iii Two of $G_{a,b}$, $G_{c,d}$, $G_{e,f}$ are infinite complex rational grey numbers, Let's suppose $G_{a,b}$, $G_{c,d}$ are finite complex rational grey numbers.

$G_{a,b} \oplus (G_{c,d} \oplus G_{e,f})$

$= G_{a,b} \oplus G_{c+e,\ d+f}$ (infinite complex rational graynumber)

$= G_{a+c+e,\ b+d+f}$ (infinite type)

$(G_{a,b} \oplus G_{c,d}) \oplus G_{e,f} = G_{a,+c,\ b+d} \oplus G_{e,f}$

$=Ga+c+e,\ b+d+f$ (infinite type)

$\therefore\ Ga, b \oplus (Gc, d \oplus Ge, f) = (Ga, b \oplus Gc, d) \oplus Ge, f$

Ⅳ Ga, b, Gc, d and Ge, f are all infinite complex rational grey numbers, We know:

$Ga, b \oplus (Gc, d \oplus Ge, f) = (-\infty, +\infty)$

$= (Ga, b \oplus Gc, d) \oplus Ge, f$

Look at the above mentioned,

$\therefore\ Ga, b \oplus (Gc, d \oplus Ge, f) = (Ga, b \oplus Gc, d) \oplus Ge, f$. TReorem6:

If Ga, b, Gc, d, Ge, f are all infinite or finite complex rational grey numbers, and a, b, c, d, e f are all non-negative or non-positive:

$Ga, b \odot (Gc, d \oplus Ge, f) = Ga, b \odot Gc, d \oplus Ga, b \odot Ge, f.$

Reference

(1) Wu. HeQin, yue chaugan, Introduction of the Grey Mathematics, HeBei People's Publishing House, 1989.

(2) Deng julong, The Grey Controlling System, Huazhoug Universitg of Science and Technology Press 1985.

(3) Wang Qingyin, wu Hegin, The Concept of Gery Number and Its Property, Proceeding of NAFIPS'88.