CONVERGENCE IN L-FUZZY PROXIMITY SPACES

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ABSTRACT

In this paper, we give the concept of L-fuzzy proximal convergence in L-fuzzy proximity spaces, and get its an alternate description. We establish the relations between L-fuzzy proximal convergence and ordinary proximal convergence, therefore, it is "good extension". Also, we disscus the basic properties of L-fuzzy proximal convergence, and obtain some results analogous to those that hold for ordinary proximity spaces.

KEY WORDS

L-fuzzy proximity spaces, L-fuzzy proximity neighborhood, L-fuzzy proximity continuous, L-fuzzy proximity product operator, L-fuzzy proximal convergence.

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1 Introduction

In /1/, Katsaras introduced and studied fuzzy proximity spaces. Later, other authors pay attention to this theory, e.g. /2/--/8/ ect. get a set of results.

In the present paper, we shall introduce and disscus the convergence in L-fuzzy proximity spaces, and show that there exists closed relations between L-fuzzy proximal convergence and ordinary proximal convergence, which is said to be "good extension" similar Lowen's, c.f. /9/. Also, we prove that L-fuzzy proximal convergence keep many basic properties.

2 Preliminaries

In this paper $L = (L, \vee, \wedge, ')$ always denotes a completely distributive lattice with order-reversing involution. Let 0 be the least element and 1 be the greatest element in L. Suppose X is a nonempty (usual) set. L^X will denote the family of all L-fuzzy set on X. It is clear that $L^X = (L^X, \vee, \wedge, ')$ is a completely distributive lattice with order-reversing involution, which has the least element $\underline{0}$ and the greatest element $\underline{1}$. $A \in L^X$, $\mathrm{Supp}A = \{x \in X: A(x) > 0\}$ is called the support of A.

Definition 2.1 (/10/) $F: L_1^{X_1} \to L_2^{X_2}$ is called an order homomorphism iff (1) F is union-preserving; (2) F^{-1} is order-reversing involution preserving, there $F^{-1}(B) = \vee\{A \in L_1^{X_1}: F(A) \le B\}$ for each $B \in L_2^{X_2}$.

Suppose $f: X_1 \to X_2$ is an ordinary mapping, it induces an order homomorphism $F: L_1^{X_1} \to L_2^{X_2}$ as follow:

$$F(A)(y) = \begin{cases} \vee\{A(x): x \in f^{-1}(y)\}, & \text{for } f^{-1}(y) \ne \phi \\ \underline{0}, & \text{otherwise} \end{cases} \quad \text{for } A \in L_1^{X_1},\ y \in X_2$$

$$F^{-1}(B) = B \circ f, \text{ for } B \in L_2^{X_2}.$$

We say that F is Zadeh's order homomorphism induced by f. There exists a 1--1 correspondence between Zadeh's order homomorphism F and its ordinary induced mapping f.

Lemma 2.2 (/10/,/11/) Let $F: L_1^{X_1} \to L_2^{X_2}$ be an order homomorphism, then: (1) $A \le F^{-1}(F(A))$ for any $A \in L_1^{X_1}$; $B \ge F(F^{-1}(B))$ for any $B \in L_2^{X_2}$; (2) $A \le F^{-1}(B)$ iff $F(A) \le B$ for any $A \in L_1^{X_1}$, $B \in L_2^{X_2}$; (3) F^{-1} is union-preserving and intersection-preserving.

Lemma 2.3 Let $F: L_1^{X_1} \to L_2^{X_2}$ be Zadeh's order homomorphism induced by f. Then $\mathrm{Supp}F(A) = f(\mathrm{Supp}A)$, for any $A \in L_1^{X_1}$.

Proof $f(\mathrm{Supp}A) = \{f(x): A(x) > 0,\ x \in X\}$

$\mathrm{Supp}F(A) = \{y: \vee\{A(x): x \in f^{-1}(y)\} > 0\}$

Firstly, $f(x) \in f(\mathrm{Supp}A)$, then $A(x) > 0$. Let $y = f(x)$, we have $\bigvee\{A(x): x \in f^{-1}(y)\} > 0$, therefore, $f(x) = y \in \mathrm{Supp}F(A)$.

Secondly, $y \in \mathrm{Supp}F(A) \Longrightarrow \bigvee\{A(x): x \in f^{-1}(y)\} > 0$, hence, there exists $x_0 \in f^{-1}(y)$, $A(x_0) > 0$, thus $y = f(x_0) \in f(\mathrm{Supp}A)$.

<u>Definition</u> 2.4 (/4/) A binary relation δ on L^X is called an L-fuzzy proximity iff

(P1) $A \delta B \Longrightarrow B \delta A$;

(P2) $A \nleq B' \Longrightarrow A \delta B$;

(P3) $A \delta B$, $A \leqslant C$, $B \leqslant D \Longrightarrow C \delta D$;

(P4) $A \delta (B \vee C) \Longrightarrow A \delta B$ or $A \delta C$;

(P5) $\underline{0}\, \bar{\delta}\, \underline{1}$;

(P6) $A \bar{\delta} B \Longrightarrow$ there exists a $C \in L^X$ such that $A \bar{\delta} C$ and $C' \bar{\delta} B$.

The pair (L^X, δ) is called an L-fuzzy proximity space.

<u>Lemma</u> 2.5 Let (L^X, δ) be an L-fuzzy proximity space, then

(1) $A \delta (B \vee C) \Longleftrightarrow A \delta B$ or $A \delta C$;

(2) $A \bar{\delta} B \Longleftrightarrow$ there exists a $C \in L^X$ such that $A \bar{\delta} C$ and $C' \bar{\delta} B$.

(3) $A_i \bar{\delta} B_i$, $i = 1,2,\ldots,n$, $\Longrightarrow$ $(\bigwedge_{i=1}^{n} A_i) \bar{\delta} (\bigvee_{i=1}^{n} B_i)$.

Proof There we shall prove (2), others are similar.

($\Longrightarrow$) It is clear from (P6).

($\Longleftarrow$) Let $A \bar{\delta} C$, by (P2), $A \leqslant C'$. Since $C' \bar{\delta} B$, hence $A \bar{\delta} B$.

<u>Definition</u> 2.6 (/4/) Let (L^X, δ) be an L-fuzzy proximity space. We define $c_\delta: L^X \to L^X$ by $c_\delta(A) = \bigwedge\{B \in L^X: A \bar{\delta} B'\}$ for any $A \in L^X$. Then c_δ is an L-fuzzy K closure operator on L^X. We Call that c_δ is the L-fuzzy K closure operator induced by δ, and τ_δ, which is defined by $\tau_\delta = \{A \in L^X: c_\delta(A') = A'\}$, is the L-fuzzy topology induced by δ.

<u>Definition</u> 2.7 Let ($L_i^{X_i}, \delta_i$), $i = 1,2$, be L-fuzzy proximity spaces. An order homomorphism F: ($L_1^{X_1}, \delta_1$) $\to$ ($L_2^{X_2}, \delta_2$) is called L-fuzzy proximity continuous iff $A \delta_1 B$ implies $F(A) \delta_2 F(B)$ for any $A, B \in L_1^{X_1}$.

<u>Definition</u> 2.8 Let $\{(L^{X_t}, \delta_t)\}_{t \in T}$ be a family of L-fuzzy proximity

spaces and $X = \prod_{t\in T} X_t$. P_t: $L^X \to L^X$ is Zadeh's projection. For $A, B \in L^X$, we define $A \delta B$ as follow: $A \delta B \Leftrightarrow$ if $A = \bigvee_{i=1}^{n} A_i (A_i \in L^X)$, $B = \bigvee_{j=1}^{m} B_j (B_j \in L^X)$, then there exist i_0 and j_0 such that $P_t(A_{i_0}) \delta_t P_t(B_{j_0})$ for each $t \in T$. Then δ is an L-fuzzy proximity on L^X. It is called the L-fuzzy proximity product operator. The pair (L^X, δ) is said to be the product of $\{(L^{X_t}, \delta_t)\}_{t\in T}$

Remark Definition 2.7 and 2.8 is generalizations of those corresponding definitions in /1/.

3 Definition of L-fuzzy proximal convergence

Suppose ($D, \leqslant$) is a directed set. OHOM($L_1^{X_1}$, $L_2^{X_2}$) denotes the family of all order homomorphism from $L_1^{X_1}$ to $L_2^{X_2}$. The mapping S: D $\to$ OHOM($L_1^{X_1}$, $L_2^{X_2}$) is called the order homomorphism net, we write $S = \{F_n: n \in D\}$.

Definition 3.1 Let ($L_2^{X_2}$, δ_2) be an L-fuzzy proximity space and $S = \{F_n: L_1^{X_1} \to L_2^{X_2} : n \in D\}$ be an order homomorphism net. S is said to converge L-fuzzy proximally to the order homomorphism $F: L_1^{X_1} \to L_2^{X_2}$ iff $F(A) \bar{\delta}_2 B$ for $A \in L_1^{X_1}$, $B \in L_2^{X_2}$ implies $F_n(A) \bar{\delta}_2 B$ eventually, or equivalency there exists a $N \in D$ such that $F_n(A) \bar{\delta}_2 B$ for all $n \geqslant N$.

Propostion 3.2 Let ($L_2^{X_2}$, δ_2) be an L-fuzzy proximity space and the order homomorphism net $S = \{F_n: L_1^{X_1} \to L_2^{X_2}: n \in D\}$ converge L-fuzzy proximally to F. Then any subnet of S converges also L-fuzzy proximally to F.

Proof It is clear from Definition 3.1 .

Definition 3.3 (/4/) Let (L^X, δ) be an L-fuzzy proximity space, A, $B \in L^X$. We called B a L-fuzzy proximity neighborhood of A iff $A \bar{\delta} B'$.

Remark It is clear that $c_\delta(A)$ is infimum of the set of all L-fuzzy proximity neighborhood of A.

Propostion 3.4 $A \bar{\delta} B$ iff there exist $\mathcal{N}_A$ and $\mathcal{N}_B$, which are respectively L-fuzzy proximity neighborhoods of A and B, such that $\mathcal{N}_A \leq \mathcal{N}_B'$.

Proof ($\Rightarrow$) If $A \bar{\delta} B$, by Lemma 2.5, there exists $C \in L^X$ such that $A \bar{\delta} C$ and $C' \bar{\delta} B$. Let $\mathcal{N}_A = C'$, $\mathcal{N}_B = C$, hence $\mathcal{N}_A \leq \mathcal{N}_B'$.

($\Leftarrow$) If there exist $\mathcal{N}_A$ and $\mathcal{N}_B$, $\mathcal{N}_A \leq \mathcal{N}_B'$, by Definition 3.3, $A \bar{\delta} \mathcal{N}_A'$, $B \bar{\delta} \mathcal{N}_B'$, hence $A \bar{\delta} \mathcal{N}_A'$, $B \bar{\delta} \mathcal{N}_A$, by Lemma 2.5, $A \bar{\delta} B$.

<u>Propostion</u> 3.5 Let ($L_2^{X_2}$, δ_2) be an L-fuzzy proximity space and $S = \{F_n: L_1^{X_1} \to L_2^{X_2}: n \in D\}$ be an order homomorphism net. Then the net S converges L-fuzzy proximally to order homomorphism $F: L_1^{X_1} \to L_2^{X_2}$ iff B is an L-fuzzy proximity neighborhood of F (A) for any $A \in L_1^{X_1}$, then B is also an L-fuzzy proximity neighborhood of F_n (A) eventually.

Proof The net S converges L-fuzzy proximally to order homomorphism F $\Leftrightarrow$ F (A) $\bar{\delta}_2$ C for any $A \in L_1^{X_1}$, $C \in L_2^{X_2}$ implies F_n (A) $\bar{\delta}_2$ C eventually $\Leftrightarrow$ Let B = C', B is an L-fuzzy proximity neighborhood of F (A), then B is also an L-fuzzy proximity neighborhood of F_n (A) eventually.

4 <u>The relations between L-fuzzy proximal convergence and ordinary proximal convergence</u>

Ordinary proximity space and ordinary proximal convergence see /12/.

<u>Lemma</u> 4.1 Let (X, μ) be an ordinary proximity space. Then the binary relation $\delta = i(\mu)$ on L^X defined by: $A \delta B$ iff there exist C and D, subset of X, such that $A \leq \chi_C$, $B \leq \chi_D$, and $C \bar{\mu} D$, is an L-fuzzy proximity. It is called the L-fuzzy proximity induced by μ . The pair (L^X, δ) is said to be the L-fuzzy proximity space induced by (X, μ).

<u>Lemma</u> 4.2 Let (L^X, δ) be an L-fuzzy proximity space. We define the binary relation $\mu = \omega(\delta)$ as follow: $C \mu D$ iff $\chi_C \delta \chi_D$ for C and D, subset of X. Then μ is an ordinary proximity on X, it is called the ordinary proximity induced by δ . The pair (X, μ) is said to be the ordinary proximity space induced by (L^X, δ).

<u>Lemma</u> 4.3 Let (L^X, δ) be an L-fuzzy proximity space and (X, μ) be an ordinary proximity space, $r = i(\mu)$ which is defined as above. Then:

(1) $ArB \Leftrightarrow \text{Supp}A \, \mu \, \text{Supp}B$ for any A, $B \in L^X$;

(2) $A \delta B \Leftrightarrow \chi_{\text{supp}A} \, \delta \, \chi_{\text{supp}B}$ for any A, $B \in L^X$.

<u>Remark</u> (1) Lemma 4.1 , 4.2 and 4.3 are generalizations of those

corresponding propostions in /1/ and /8/.

(2) For i defined by Lemma 4.1 and ω defined by Lemma 4.2, we have that $i\circ\omega$ is the identically mapping from LFP to itself, there LFP denotes the collection of all L-fuzzy proximity on L^X.

Propostion 4.4 Let (L^X, δ) be the L-fuzzy proximity space induced by the ordinary proximity space (X, μ). Then Zadeh's order homomorphism net $S = \{F_n: L_1^Y \to L^X: n \in D\}$ converges L-fuzzy proximally to Zadeh's order homomorphism F in (L^X, δ) iff the net $S^* = \{f_n: Y \to X: n \in D\}$ converges proximally to f in (X, μ). There f_n and f are respecturely ordinary induced mapping of Zadeh's order homomorphism F_n and F, they are determined uniquely by F_n and F.

Proof ($\Longrightarrow$) Let $f(C)\bar{\mu}D$ for any $C\subset Y$, $D\subset X$. Since $f(C) = \mathrm{Supp}\chi_{f(C)}$, $D = \mathrm{Supp}\chi_D$, we have $\mathrm{Supp}\chi_{f(C)}\bar{\mu}\,\mathrm{Supp}\chi_D$, by Lemma 4.3 (1), $\chi_{f(C)}\bar{\delta}\chi_D$, by $F(\chi_C) = \chi_{f(C)}$, $F(\chi_C)\bar{\delta}\chi_D$.

By L-fuzzy proximal convergence of S, $F_n(\chi_C)\bar{\delta}\chi_D$ eventually, hence, $\chi_{f_n(C)}\bar{\delta}\chi_D$ eventually. By Lemma 4.3 (1), $\mathrm{Supp}\chi_{f_n(C)}\bar{\mu}\,\mathrm{Supp}\chi_D$ eventually, therefore, $f_n(C)\bar{\mu}D$ eventually.

($\Longleftarrow$) Let $F(A)\bar{\delta}B$ for any $A\in L_1^Y$, $B\in L^X$. By Lemma 4.3 (1), $\mathrm{Supp}F(A)\bar{\mu}\,\mathrm{Supp}B$, by Lemma 2.3, $f(\mathrm{Supp}A)\bar{\mu}\,\mathrm{Supp}B$.

Since S^* converges proximally to f, we have $f_n(\mathrm{Supp}A)\bar{\mu}\,\mathrm{Supp}B$ eventually. Hence, $\mathrm{Supp}F_n(A)\bar{\mu}\,\mathrm{Supp}B$ eventually, therefore, $F_n(A)\bar{\delta}B$ eventually.

Propostion 4.5 Let (X, μ) be the ordinary proximity space induced by the L-fuzzy proximity space (L^X, δ). Then the mapping net $S^* = \{f_n: Y \to X: n \in D\}$ converges proximally to the mapping f in (X, μ) iff the Zadeh's order homomorphism net $S = \{F_n: L_1^Y \to L^X: n \in D\}$ converges L-fuzzy proximally to the Zadeh's order homomorphism F. There F_n and F are respecturely the Zadeh's order homomorphism induced by f_n and f.

Proof ($\Longrightarrow$) Let $F(A)\bar{\delta}B$ for any $A\in L_1^Y$, $B\in L^X$. By Lemma 4.3 (2), $\chi_{\mathrm{Supp}F(A)}\bar{\delta}\chi_{\mathrm{Supp}B}$, by Lemma 2.3, $\chi_{f(\mathrm{Supp}A)}\bar{\delta}\chi_{\mathrm{Supp}B}$, hence,

$f\,(\mathrm{Supp}A)\,\bar{\mu}\,\mathrm{Supp}B$.

Since S* converges proximally to f, we have $f_n\,(\mathrm{Supp}A)\,\bar{\mu}\,\mathrm{Supp}B$ eventually. Hence, $\mathrm{Supp}(F_n(A))\,\bar{\mu}\,\mathrm{Supp}B$ eventually, by Lemma 4.2, $\chi_{\mathrm{Supp}F_n(A)}\,\bar{\delta}\,\chi_{\mathrm{Supp}B}$ eventually, therefore, $F_n\,(A)\,\bar{\delta}\,B$ eventually.

($\Leftarrow$) Let $f\,(C)\,\bar{\mu}\,D$ for any $C\subset Y$, $D\subset X$. By Lemma 4.2, $\chi_{f(C)}\,\bar{\delta}\,\chi_D$, hence $F\,(\chi_C)\,\bar{\delta}\,\chi_D$.

Since the L-fuzzy proximal convergence of S, $F_n\,(\chi_C)\,\bar{\delta}\,\chi_D$ eventually, hence $\chi_{f_n(C)}\,\bar{\delta}\,\chi_D$ eventually, by Lemma 4.2, $f_n\,(C)\,\bar{\mu}\,D$ eventually.

Remark By Propostion 4.4 and 4.5, we may say that the L-fuzzy proximal convergence is "good extension" similar Lowen's, c.f. /9/.

5 The basic properties of L-fuzzy proximal convergence

Propostion 5.1 Let $(L_i^{X_i}, \delta_i)$, i = 1, 2, be L-fuzzy proximity spaces and the continuous order homomorphism net $S=\{F_n: (L_1^{X_1}, \delta_1)\to(L_2^{X_2}, \delta_2)$ F_n is L-fuzzy proximity continuous, $n\in D\}$ converge L-fuzzy proximally to the order homomorphism $F: (L_1^{X_1}, \delta_1)\to(L_2^{X_2}, \delta_2)$. Then F is also L-fuzzy proximity continuous.

Proof Let $A\,\delta_1\,B$ for any $A, B\in L_1^{X_1}$, we shall prove $F\,(A)\,\delta_2\,F\,(B)$. Otherwisely, suppose $F\,(A)\,\bar{\delta}_2\,F\,(B)$, by Lemma 2.5 (2), there exists $C\in L_2^{X_2}$ such that $F\,(A)\,\bar{\delta}_2\,C$ and $C'\,\bar{\delta}_2\,F\,(B)$. Since S converges L-fuzzy proximally to F, there exists a common $N\in D$ such that $F_n\,(A)\,\bar{\delta}_2\,C$ and $C'\,\bar{\delta}_2\,F_n\,(B)$ for all $n\geqslant N$. Hence, $F_n\,(A)\,\bar{\delta}_2\,F_n\,(B)$ for all $n\geqslant N$, contradicting the conditions that each F_n is L-fuzzy proximity continuous. Therefore, F is L-fuzzy proximity continuous.

Propostion 5.2 Let $(L_1^{X_1}, \tau)$ be an L-fuzzy topological space and $(L_2^{X_2}, \delta)$ be an L-fuzzy proximity space. If the order homomorphism net $S=\{F_n: (L_1^{X_1}, \tau)\to(L_2^{X_2}, \tau_\delta): F_n$ is L-fuzzy continuous, $n\in D\}$ converges L-fuzzy proximally to the order homomorphism F in $(L_2^{X_2}, \delta_2)$, then $F: (L_1^{X_1}, \tau)\to(L_2^{X_2}, \tau_\delta)$ is also L-fuzzy continuous.

Proof It is simial to Propostion 5.1 and hence omitted.

<u>Propostion</u> 5.3 Let (L_2^Y, δ) be the product of the collection $\{(L_2^{Y_t}, \delta_t)\}_{t\in T}$ of L-fuzzy proximity spaces. Then the order homomorphism net $S = \{F_n: L_1^X \to L_2^Y: n\in D\}$ converges L-fuzzy proximally to F in (L_2^Y, δ) iff for each $t\in T$, the order homomorphism net $S_t = \{P_t\circ F_n: L_1^X \to L_2^{Y_t}: n\in D\}$ converges L-fuzzy proximally to $P_t\circ F$ in ($L_2^{Y_t}, \delta_t$).

Proof ($\Longrightarrow$) For $A\in L_1^X$, $B\in L_2^{Y_t}$, there exists $D\in L_2^Y$ such that $B = P_t(D)$. Let $P_t\circ F(A)\,\bar{\delta}_t\,P_t(D)$ for each $t\in T$, we shall prove $P_t\circ F_n(A)\,\bar{\delta}_t\,P_t(D)$ eventually.

If $A = \bigvee_{i=1}^{n} A_i (A_i\in L_1^X)$, $D = \bigvee_{j=1}^{m} D_j (D_j\in L_2^Y)$, then $P_t\circ F_n(A)\,\bar{\delta}_t\,P_t(D)$ iff $P_t\circ F_n(A_i)\,\bar{\delta}_t\,P_t(D_j)$ for each i and j.

Now suppose there exist i_0 and j_0, such that $P_t\circ F_n(A_{i_0})\,\delta_t\,P_t(D_{j_0})$ frequently. By Definition 2.8, $F_n(A)\,\delta\,D$ frequently. Since S converges L-fuzzy proximally to F, hence we have $F(A)\,\delta\,D$, by L-fuzzy proximity continuous of P_t, $P_t\circ F(A)\,\delta_t\,P_t(D)$ for each $t\in T$. This is a constradiction, therefore S_t converges L-fuzzy proximally to $P_t\circ F$.

($\Longleftarrow$) Let $F(A)\,\bar{\delta}\,B$ for each $A\in L_1^X$, $B\in L_2^Y$, we shall prove $F_n(A)\,\bar{\delta}\,B$ eventually.

Now suppose $F_n(A)\,\delta\,B$ frequently. Since P_t is L-fuzzy proximity continuous, we have $P_t\circ F_n(A)\,\delta_t\,P_t(B)$ frequently. By the L-fuzzy proximal convergence of S_t for each $t\in T$, $P_t\circ F(A)\,\delta_t\,P_t(B)$, hence $(\bigvee_{i=1}^{n} P_t\circ F(A_i))\,\delta_t\,(\bigvee_{j=1}^{m} P_t(B_j))$ for each $t\in T$, by Lemma 2.5 (1), for each $t\in T$, there exist i_0 and j_0 such that $P_t\circ F(A_{i_0})\,\delta_t\,P_t(B_{j_0})$, thus by Definition 2.8, $F(A)\,\delta\,B$. This is a constradiction, therefore S converges L-fuzzy proximally to F. This completes the proof.

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