INVARIANT SUBSPACE OF FUZZY VECTOR SPACE V_n

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ABSTRACT

In this paper we studied the invariant subspace of fuzzy vector space V_n, and we gave its decition theorem.

Keywords: Fuzzy vector space V_n, Transformation space of fuzzy vector space V_n, Invariant subspace of fuzzy vector space V_n.

FOREWORD

The transformation is important concept in mathematics. In the document [1], it was led into fuzzy mathematics: We gave a fuzzy matrix $R = (r_{ij})_{n \times m}$ and a fuzzy vector $A = (a_1 \ldots a_n)$. $B = A \circ R$ is called the fuzzy transformation. R is called a fuzzy transformation matrix or is abbreviated fuzzy transformation or transformation.

In this paper we preliminarily discussed the transformation space of fuzzy vector space V_n, the invariant subspace of fuzzy vector space V_n and gave its dicition theorem.

In this paper we used the symbols and definitions in the documents [2,3] .

1 LINEAR TRANSFORMATION OF FUZZY VECTOR SPACE V_n

Theorem 1.1 Let $A, B \in V_n$, $R \in \mathcal{M}_{n \times m}$. Then

(1) $(A \cup B) \circ R = (A \circ R) \cup (B \circ R)$;

(2) $(kA) \circ R = k(A \circ R)$, $k \in [0,1]$.

Therefore the arbitrary fuzzy transformation is a linear .

Theorem 1.2 Let $A_1, \ldots, A_t \in V_n$ are the vector set of a linear dependence, and $R \in \mathcal{M}_{n \times m}$ is a fuzzy transformation of V_n. Then $A_1 \circ R, \ldots, A_t \circ R$ are also a linear dependence.

Theorem 1.3 Let $A_1, \ldots, A_t \in V_n$ are the set of a linear independence, and $R \in \mathcal{M}_{n \times n}$ is a invertible. Then $A_1 \circ R, \ldots, A_t \circ R$ are a liearly independent. (The definition of a invertible fuzzy matrix see [3]).

Theorem 1.4 Let W is a subspace of V_n, and $R\in\mathcal{M}_{n\times m}$ is a fuzzy transformation of W. Then W R is a subspace of W, where $W\circ R = \{ B \mid \forall A\in W, A\circ R = B \}$. $W\circ R$ is called a transformation of W.

Theorem 1.5 Let $A_1, \ldots, A_t \in V_n$ and $R\in\mathcal{M}_{n\times m}$. If $A_1\circ R, \ldots, A_t\circ R$ are a linearly independent, then $A_1, \ldots, A_t$ are also linearly independent.

2 THE INVARIANT SUBSPACE OF FUZZY VECTOR SPACE V_n

Definition 2.1 Let W is a subspace of V_n, and $R\in\mathcal{M}_{n\times n}$ is a transformation of W. If $W\circ R \subseteq W$, then W are called a invariant subspace of R.

Example 2.1 For arbitrary $R\in\mathcal{M}_{n\times n}$. Then

(1) $\{\theta\}$ is a invariant subspace of R, where θ is the fuzzy zero vector in V_n.

(2) V_n is a invariant subspace of R.

Theorem 2.1 Let both W_1 and W_2 are subspace of V_n, and $R\in\mathcal{M}_{n\times n}$. If both W_1 and W_2 are the invariant subspace of R, then

(1) $W_1 \cup W_2$ is a invariant subspace of R;

(2) $W_1 \cap W_2$ is a invariant subspace of R.

Theorem 2.2 Let $R\in\mathcal{M}_{n\times n}$ is a transformation of V_n. We define that

$$W(\delta) \triangleq \{ A \mid A = (a_1 \ldots a_n), \forall a_i \le \delta \ , \ \delta\in[0,1] \} .$$

Then $W(\delta)$ is a invariant subspace of R.

Theorem 2.3 Let a subspace W of V_n is a invariant subspace of both transformation $R\in\mathcal{M}_{n\times n}$ and $T\in\mathcal{M}_{n\times n}$. Then

(1) W is a invariant subspace of $R \cup T$;

(2) $W\circ R$ is a invariant subspace of R;

(3) $W\circ R$ is a invariant subspace of T;

(4) W is a invariant subspace of R^m, where m is an arbitrary positive integer;

(5) W is a invariant subspace of αR, where $\alpha\in[0,1]$;

(6) Let $f(R) = \bigcup_{k=0}^{m} \alpha_k R^k$, where $\alpha_i \in [0,1]$, $(i=0,1, \ldots, m)$, $R^0 = I_n$. Then W is a invariant subspace of f(R).

Theorem 2.4 Let a finitely generated subspace W of V_n is a subspace of R, R is a invertible, and $A_1, \ldots, A_t$ are a basis of W. Then

(1) $A_1\circ R, \ldots, A_t\circ R$ are a basis of W;

(2) W is a invariant subspace of R^{-1}.

Theorem 2.5 (The decition theorem of the invariant subspace of fuzzy vector space V_n) Let $W = L(A_1,\ldots,A_t)$, $\forall A_i \in V_n$. Then W is a invariant subspace of R if and only if $A_1 \bullet R, \ldots, A_t \bullet R \in W$.

Theorem 2.6[4] Let $R = (r_{ij})_{n\times n}$ is a transformation of V_n, and let

$$r_0 = \bigwedge_j \left(\bigvee_i r_{ij} \right), \quad W = \{ A \mid A=(\mu \ldots \mu),\ \mu \leq r_0 \}.$$

Then W is invariant subspace of R.

Theorem 2.7 Let W is a finitely generated subspace of V_n, and $R = aI_n$, $a \in [0,1]$. Then $W \bullet R$ is a invariant subspace of R.

Theorem 2.8 Let W is a finitely generated subspace of V_n, and $R = (r_{ij})_{n\times n}$. If $A_i = (a_{i1} \ldots a_{in})$, $(i=1, \ldots, t)$ are a basis of W, and evrey element of R satisfies

$$r_{ij} \geq \max_{i,j} \{ a_{ij} \}, \quad 1 \leq i,j \leq n.$$

Then W is a invariant subspace of R.

REFERENCE

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