FUZZY LOGIC ASSOCIATION PRINCIPLE

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Abstract: An association principle is analyzed and synthetized by means of fuzzy logic. Dynamical and statical model of fuzzy logic neural network association is considered.

1. Introduction.

In [1], [2], [3], and [4] an unified theory of fuzzy logic neural systems has been introduced.

We intend to apply this theory to analyze and synthetise of an association process.

An association can be defined as the process of forming mental connections or bonds between sensations, ideas, or memories.

We impose a hypothesis that the association process is inherently uncertain. Dynamical and statical models of fuzzy logic neural associations are the subject of this paper.

Fuzzy logic theory is used as a formal tool to attack a vague nature of the association process.

2. Fuzzy logic neural association principle.

Let us consider a neuron depicted in Figure 1.

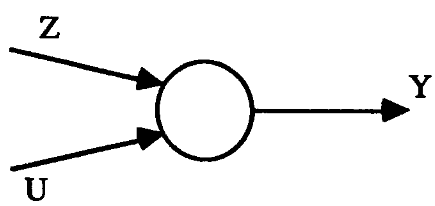

Figure 1. Single Fuzzy neuron

Let the neuron be trained by the following set of fuzzy rules

$$\{\text{IF } Z_{(i)} \text{ and } U_{(i)} \text{ then } Y_{(i)}, \text{ ALSO}\}, i = 1, 2, 3, \dots, I \qquad (1)$$

where $Z_{(i)}$ and $U_{(i)}$ are input fuzzy variables, and $Y_{(i)}$ is fuzzy output variable.

A formal behavior of the above neuron can be expressed as

$$Y = U \circ R_1 \wedge Z \circ R_2 \tag{2}$$

where U and Z are current fuzzy inputs, and Y is a present output; o stands for max-min composition; $\wedge$ means min operator.

Fuzzy relations R_1, and R_2 are defined as

$$R_1 = \bigvee_i \{ U_{(i)} \wedge Y_{(i)} \}, \text{ and}$$

$$R_2 = \bigvee_i \{ Z_{(i)} \wedge Y_{(i)} \}.$$

Now we formulate an association principle. Given (2), find input signal U which strictly associates signal Y with signal Z, i.e. $Y = Z$.

In view of (2) and the above principle we get

$$Z = U \circ R_1 \wedge Z \circ R_2 \tag{3}$$

We call Equation (3) a fuzzy association equation.

To resolve the above association principle we have to:

1) given R_1, R_2 and Z, find U satisfies (3)

2) given Z, U, find R_1 and R_2 satisfy (3)

From practical point of view it is more convenient to formulate similarity association principle:

Given (1), find input signal U which similarly associates signal Y with signal Z, i .e.

$Y = Z \circ S$, where S is given similarity matrix.

In view of (2), the fuzzy association equation (3) can be rewritten as

$$Z \circ S = U \circ R_1 \wedge Z \circ R_2 \qquad (4)$$

To resolve the similarity association principle we have to

1) given S, R_1, R_2 and Z, find U satisfies (4).

2) given $Z, U,$ and S, find R_1 and R_2 satisfy (4).

3) given $R_1, R_2,$ and Z, U, find S satisfies (4).

Let us consider fuzzy logic dynamic neuron depicted in Figure 2.

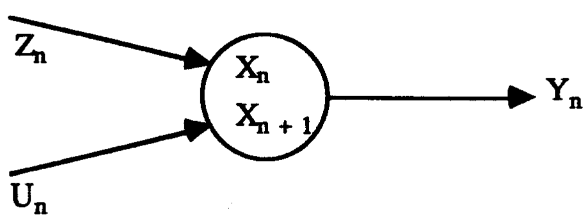

Figure 2. Fuzzy dynamic neuron

Let the neuron be trained by the following set of fuzzy rules:

$$\{\text{IF } X_{n(i)} \text{ and } Z_{n(i)} \text{ and } U_{n(i)} \text{ then } Y_{n+1(i)} \text{ and } Y_{n(i)}, \text{ ALSO}\} \qquad (5)$$

where $i = 1, 2, 3, \dots, I$ is the number of rules and $n = 0, 1, 2, 3, \dots$ is a discrete-time variable.

X_n is a past memory of the neuron, and X_{n+1} is an intended memory of the neuron; Z_n,

U_n are current inputs, and Y_n is present output.

A dynamic behavior of the above neuron can be now expressed as

$$\left.\begin{aligned} X_{n+1} &= X_n \circ R_1 \wedge U_n \circ R_2 \wedge Z_n \circ R_3 \quad &(a) \\ Y_n &= X_n \circ R_4 \wedge U_n \circ R_5 \wedge Z_n \circ R_6 \quad &(b) \end{aligned}\right\} \quad (6)$$

where

$$R_1 = \bigvee_i \{ X_{n(i)} \wedge X_{n+1(i)} \}$$

$$R_2 = \bigvee_i \{ U_{n(i)} \wedge X_{n+1(i)} \}$$

$$R_3 = \bigvee_i \{ Z_{n(i)} \wedge X_{n+1(i)} \}$$

$$R_4 = \bigvee_i \{ X_{n(i)} \wedge Y_{n\,(i)} \}$$

$$R_5 = \bigvee_i \{ U_{n(i)} \wedge Y_{n\,(i)} \}$$

$$R_6 = \bigvee_i \{ Z_{n(i)} \wedge Y_{n\,(i)} \}$$

Now we may formulate a dynamic association principle:

Given (6), find input signals U_n which strictly associates signal Y_n with signal Z_n, i.e. $Y_n = Z_n$.

In view of (6) and the above principle we get

$$\left.\begin{aligned} X_{n+1} &= X_n \circ R_1 \wedge U_n \circ R_2 \wedge Z_n \circ R_3 \\ Z_n &= X_n \circ R_4 \wedge U_n \circ R_5 \wedge Z_n \circ R_6 \end{aligned}\right\} \quad \begin{matrix}(a)\\(7)\\(b)\end{matrix}$$

Solving (7)(a) we get [1]

$$X_n = X_0 \circ R_1^n \wedge \bigwedge_{i=0}^{n-1} U_i \circ R_2 \circ R_1^{n-i-1} \wedge \bigwedge_{i=0}^{n-1} Z_i \circ R_3 \circ R_1^{n-i-1} \tag{8}$$

Substituting (8) into (7)(b) we have

$$\begin{aligned} Z_n = X_0 \circ R_1^n \circ R_4 \wedge \bigwedge_{i=0}^{n-1} U_i \circ R_2 \circ R_1^{n-i-1} \circ R_4 \\ \wedge U_n \circ R_5 \wedge \bigwedge_{i=0}^{n-1} Z_i \circ R_3 \circ R_1^{n-i-1} \wedge Z_n \circ R_6 \end{aligned} \tag{9}$$

To resolve the dynamic association principle we have to

1) given $R_1, \dots, R_6$ and X_0, for Z_n given, find U_n satisfies (9).

2) given X_0, Z_n, U_n, find $R_1, \dots, R_6$ satisfy (9).

We call equation (9) the fuzzy dynamic association equation.

We may also shape a similarity association principle for a dynamic system (6). Given (6), find input signal U_n which similarly associates signals Y_n with signal Z_n, i.e. $Y_n = Z_n \circ S$, where S is a given similarity matrix.

In view of the above principle we get

$$Z_n \circ S = X_0 \circ R_1^n \circ R_4 \wedge \bigwedge_{i=0}^{n-1} U_i \circ R_2 \circ R_1^{n-i-1} \circ R_4 \wedge U_n \circ R_5 \wedge \bigwedge_{i=0}^{n-1} Z_i \circ R_3 \circ R_1^{n-i-1} \wedge Z_n \circ R_6 \quad (10)$$

Equation (10) is the fuzzy dynamic similarity equation.

3. Fuzzy logic neural network association.

Let us consider a fuzzy logic neural network association shown in Figure 3.

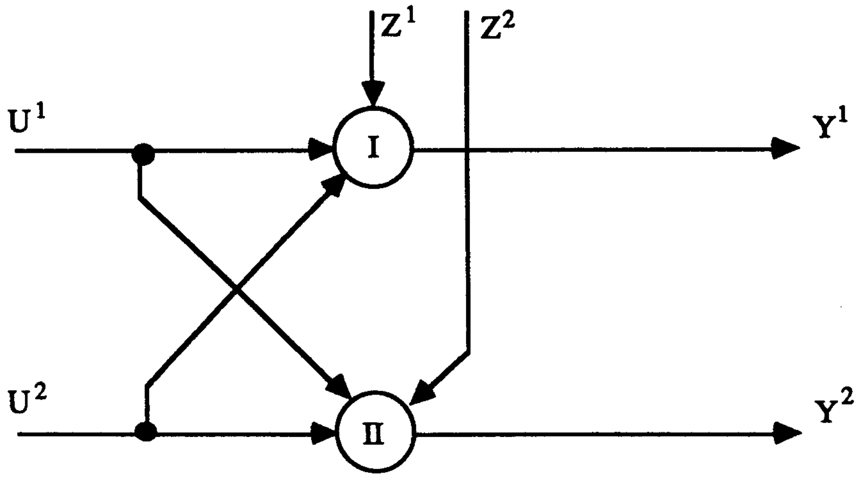

Figure 3. Fuzzy network association

Let the following set of training rules be given.

For neuron No. I

$$\{ \text{IF } U^1_{(i)} \text{ and } U^2_{(i)} \text{ and } Z^1_{(i)} \text{ then } Y^1_{(i)}, \text{ALSO}\} \tag{11}$$

For neuron No. II

$$\{ \text{IF } U^1_{(i)} \text{ and } U^2_{(i)} \text{ and } Z^2_{(i)} \text{ then } Y^2_{(i)}, \text{ALSO}\} \tag{12}$$

Fuzzy equations shaping a behavior of the network can be stated as

$$\left.\begin{aligned} Y^1 &= U^1 \circ R_1 \wedge U^2 \circ R_2 \wedge Z^1 \circ R_3 \\ Y^2 &= U^1 \circ R_4 \wedge U^2 \circ R_5 \wedge Z^2 \circ R_6 \end{aligned}\right\} \tag{13}$$

where

$$R_1 = \bigvee_i \left\{ U^1_{(i)} \wedge Y^1_{(i)} \right\}$$

$$R_6 = \bigvee_i \left\{ Z^1_{(i)} \wedge Y^2_{(i)} \right\}$$

Now we may specify a net association principle:

Given (13), find input signals U^1, U^2 which similarly associate signals Y^1, Y^2 with signals Z^1, Z^2, i.e.

$$\begin{bmatrix} Y^1 \\ Y^2 \end{bmatrix} = [Z^1 \; Z^2] \circ \begin{bmatrix} S_1 & \\ & S_2 \end{bmatrix} \qquad (14)$$

where S_1 , S_2 are given similarity matrices.

In the presence of (13) and (14) we have

$$\left.\begin{aligned} Z^1 \circ S_1 &= U^1 \circ R_1 \wedge U^2 \circ R_2 \wedge Z^1 \circ R_3 \\ Z^2 \circ S_2 &= U^1 \circ R_4 \wedge U^2 \circ R_5 \wedge Z^2 \circ R_6 \end{aligned}\right\} \qquad (15)$$

To resolve the net fuzzy association principle we have to:

1) given S_1, S_2; $R_1, \dots, R_6$ and Z_1, Z_2 find U^1, U^2 satisfy (15).

2) given S_1, S_2; Z_1, Z_2, and U^1, U^2, find $R_1, \dots, R_6$ satisfy (15).

Let as regard a fuzzy dynamic neural network association depicted in Figure 4.

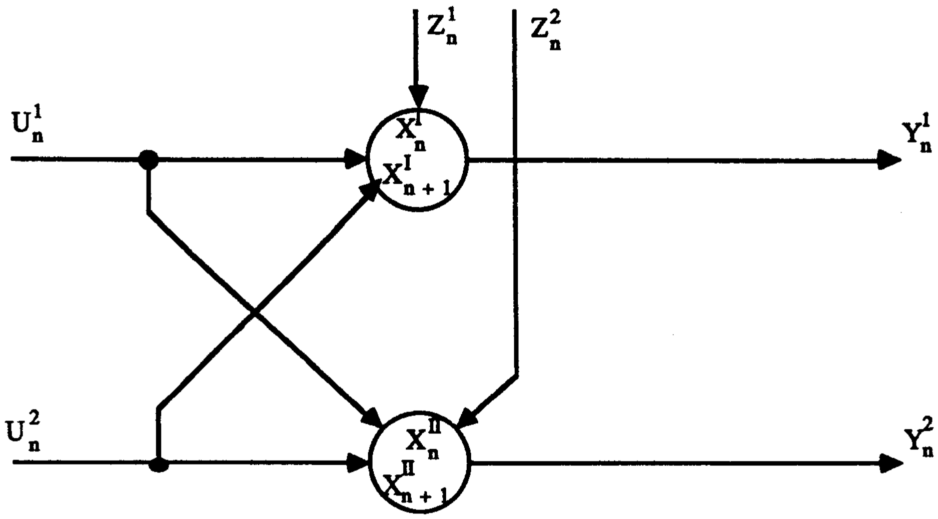

Figure 4. Fuzzy dynamic network association.

Let the considered network be trained by the following set of fuzzy rules:

For neuron No. I

$$\left\{ \text{IF } X^{I}_{n(i)} \text{ and } U^{1}_{n(i)} \text{ and } U^{2}_{n(i)} \text{ and } Z^{1}_{n(i)} \text{ then } X^{I}_{n+1(i)} \text{ and } Y^{1}_{n(i)}, \text{ ALSO} \right\} \tag{16}$$

For neuron No. II

$$\left\{ \text{IF } X^{II}_{n(i)} \text{ and } U^{1}_{n(i)} \text{ and } U^{2}_{n(i)} \text{ and } Z^{2}_{n(i)} \text{ then } X^{II}_{n+1(i)} \text{ and } Y^{2}_{n(i)}, \text{ ALSO} \right\} \tag{17}$$

Using (16) and (17) we may precise the following set of fuzzy equations:

For neuron No. I

$$\left.\begin{aligned} X^{I}_{n+1} &= X^{I}_{n} \circ R^{I}_{1} \wedge U^{1}_{n} \circ R^{I}_{2} \wedge U^{2}_{n} \circ R^{I}_{3} \wedge Z^{1}_{n} \circ R^{I}_{4} \\ Y^{1}_{n} &= X^{I}_{n} \circ R^{I}_{5} \wedge U^{1}_{n} \circ R^{I}_{6} \wedge U^{2}_{n} \circ R^{I}_{7} \wedge Z^{1}_{n} \circ R^{I}_{8} \end{aligned}\right\} \tag{18}$$

where

$$R_1^I = \bigvee_i \left\{ X_{n(i)}^I \wedge X_{n+1(i)}^I \right\}$$

$$R_8^I = \bigvee_i \left\{ Z_{n(i)}^1 \wedge Y_{n(i)}^1 \right\}$$

For neuron No. II

$$\left.\begin{aligned} X_{n+1}^{II} &= X_n^{II} \circ R_1^{II} \wedge U_n^1 \circ R_2^{II} \wedge U_n^2 \circ R_3^{II} \wedge Z_n^2 \circ R_4^{II} \\ Y_n^2 &= X_n^{II} \circ R_5^{II} \wedge U_n^1 \circ R_6^{II} \wedge U_n^2 \circ R_7^{II} \wedge Z_n^2 \circ R_8^{II} \end{aligned}\right\} \qquad (19)$$

where

$$R_1^{II} = \bigvee_i \left\{ X_{n(i)}^{II} \wedge X_{n+1(i)}^{II} \right\}$$

$$R_8^{II} = \bigvee_i \left\{ Z_{n(i)}^2 \wedge Y_{n(i)}^2 \right\}$$

Equations (18) and (19) portray the fuzzy dynamic network association of Figure 4. A dynamic net association principle may be specify as follows:

Given (18) and (19), find input signals U_n^1, U_n^2 which similarly associate signals Y_n^1, Y_n^2 with signals Z_n^1, Z_n^2, i.e.

$$\begin{bmatrix} Y_n^1 \\ Y_n^2 \end{bmatrix} = \begin{bmatrix} Z_n^1 & Z_n^2 \end{bmatrix} \circ \begin{bmatrix} S_1 & \\ & S_2 \end{bmatrix} \qquad (20)$$

where S_1, S_2 are given similarity matrices.

In view of (18), (19) and (20) we have

$$\begin{aligned} Z_n^I \circ S_1 &= X_0^I \circ R_1^I \circ R_5^I \wedge \bigwedge_{i=0}^{n-1} U_i^1 \circ R_2^I \left\{ R_1^I \right\}^{n-i-1} \circ \\ &= R_5^I \wedge U_n^1 \circ R_6^I \wedge \bigwedge_{i=0}^{n-1} U_i^2 \circ R_3^I \circ \\ &\circ \left\{ R_1^I \right\}^{n-i-1} \circ R_5^I \wedge U_n^2 \circ R_7^I \wedge \\ &\bigwedge_{i=0}^{n-1} Z_i^1 \circ R_4^I \circ \left\{ R_1^I \right\}^{n-i-1} \circ R_5^I \wedge Z_n^1 \circ R_8^I \end{aligned} \qquad (21)$$

and

$$\begin{aligned} Z_n^{II} \circ S_2 &= X_0^{II} \circ R_1^{II} \circ R_5^{II} \wedge \bigwedge_{i=0}^{n-1} U_i^1 \circ R_2^{II} \left\{ R_1^{II} \right\}^{n-i-1} \circ \\ &= R_5^{II} \wedge U_n^1 \circ R_6^{II} \wedge \bigwedge_{i=0}^{n-1} U_i^2 \circ R_3^{II} \circ \left\{ R_1^{II} \right\}^{n-i-1} \\ &\circ R_5^{II} \wedge U_n^2 \circ R_7^{II} \wedge \bigwedge_{i=0}^{n-1} Z_i^1 \circ R_4^{II} \circ \\ &\circ \left\{ R_1^{II} \right\}^{n} \circ R_5^{II} \wedge Z_n^1 \circ R_8^{II} \end{aligned} \qquad (22)$$

Equations (21) and (22) condition the similarity association principle. To resolve the above association principle we have to:

1) given X_0^I, X_0^{II}, and $R_1^I, \ldots, R_8^{II}$, and S_1, S_2, and Z_n^I, Z_n^{II}, find U_n^1, U_n^2 satisfy (21) and (22)

In the same manner we may state a memory net association principle.

Given (18), (19) and Z_n^1, Z_n^2, find input signals U_n^1, U_n^2 which similarly associate the intended and past experience of the network of Figure 4, i.e.
$X_{n+1}^{II} = X_{n+1}^I \circ S_1$, and $X_n^{II} = X_n^I \circ S_2$.

In the presence of (18) and (19) the memory association principle can be stated as follows:

$$X_n^I \circ R_1^{II} \circ S_2 \wedge U_n^1 \circ R_2^{II} \wedge U_n^2 \circ R_3^{II} \wedge Z_n^2 \circ R_4^{II} =$$

$$= X_n^I \circ R_1^I \circ S_1 \wedge U_n^1 \circ R_2^I \circ S_1 \wedge U_n^2 \circ R_3^I \circ S_1$$

$$\wedge Z_n^1 \circ R_4^I \circ S_1 \quad (23)$$

To resolve the memory net association principle we have to

1) given X_n^I, Z_n^1, Z_n^2; $R_1^I, \ldots, R_4^{II}$, and S_1, S_2, find U_n^1, U_n^2 satisfy (23)

2) given $X_n^I, Z_n^1, Z_n^2, R_1^I, \ldots, R_4^{II}$; and U_n^1, U_n^2, find S_1, S_2 satisfy (23).

4. Summary.

The association process has been structured by means of fuzzy logic.

To resolve the fuzzy logic association principle a set of fuzzy association equations must be solved.

The idea outline in this paper may be utilized in pattern recognition, machine translation, speach processing, etc.

References.

[1] M.M. Gupta, J.B. Kiszka, and G.M. Trojan, Multivariable structure of fuzzy control systems, IEEE Trans. on SMC, Vol. SMC-16, No. 5, Sept./Oct. 1986.

[2] J.B. Kiszka, and M.M. Gupta, Fuzzy logic model of single neuron, BUSEFAL, 1989.

[3] J.B. Kiszka, and M.M. Gupta, Fuzzy Logic neural network, BUSEFAL, 1989.

[4] J.B. Kiszka, and M.M. Gupta, Fuzzy logic neural network with feedback, BUSEFAL, 1989.

[5] J.B. Kiszka, and M.M. Gupta, Fuzzy logic neural processor, BUSEFAL, 1989.