BOTTOM-UP INFERENCES USING FUZZY REASONING

Kumar S. Ray

Electronics and Communication Sciences Unit
Indian Statistical Institute
203 B.T. Road, Calcutta 700035
INDIA.

Introduction

We consider the extension of the resolution principle, based on refutation, from binary logic to fuzzy logic. The extension depends on the concept of linguistic variable, compositional rules of inference, approximate reasoning and plausible reasoning. We initially consider the fuzzy resolution principle for the propositional logic and then extend it to first order logic. The completeness of the fuzzy resolution principle is discussed. The present fuzzy resolution principle provides a powerful tool for logic programming in uncertain environment.

Fuzzy logic and formulas

Fuzzy logic is an algebraic system $<[0,1], \wedge, \vee, \sim,$ where the closed interval $[0,1]$ is a set of truth values. The logical truth values are derived from the concept of multiple-valued logic [5][6]. This concept can be extended to the linguistic truth values for the fuzzy linguistic variables.

The logical operations AND ($\wedge$), OR ($\vee$) and NOT ($\sim$) are defined as follows :

$$A \wedge B = \min(A,B)$$
$$A \vee B = \max(A,B)$$
$$\sim A = 1-A \;,\; A,B \in [0,1]$$

We shall assume that a fuzzy function is a function of variables/linguistic variables $x_1, x_2, \ldots, x_n$ each of which assumes values in the closed interval [0,1].

<u>Definition 1 :</u> Fuzzy formulas are defined recursively as follows ;

(i) A variable x_i is a fuzzy formula

(ii) If F is a fuzzy formula then $\sim F$ is also a fuzzy formulaa

(iii) If F and G are fuzzy formulas then $F \wedge G$ and $F V G$ are fuzzy formulas,

(iv) The above are the only fuzzy formulas.

<u>Definition 2 :</u> A variable x_i $(i=1,\ldots,n)$ or its negation $\sim x_i$ is said to to be a literal and x_i and $\sim x_i$ are said to be complements of each other or a pair of complementary variable.

<u>Definition 3 :</u> A clause is a disjunction of literals or is a formula consisting or OR (V) of some literals.

<u>Definition 4 :</u> When a clause contains no literal, we call it the empty clause.

Given an interpretation I, the truth value of a clause C is determined uniquely by substituting a value of the closed interval [0,1] determined by the interpretation I for each variable of the clause.

Denoting the truth value assigned to x_i by $T(x_i)$, the truth value of a fuzzy formulas is T(F) and the truth value of a fuzzy clause C is T(C).

If binary logic is used in a theorem proving or problem solving system, we store a statement A instead of $\sim$ A, if the truth value of A is 1. In case the truth value of a statement A is 0, we simply store $\sim$ A. In fuzzy logic we should store a statement A instead of $\sim$ A, if the truth value of A is greater than or equal to that of $\sim$ A. That is we store A if $T(A) \geqslant 1-T(A)$. In this case $T(A) \geqslant .5$.

We can use this concept to define satisfiability in fuzzy logic as follows;

<u>Definition 5 :</u> An interpretation I is said to satisfy a formula F if $T(F) \geqslant 0.5$. An interpretation I is said to falsify F if $T(F) \leqslant 0.5$. If $T(F)=0.5$ under I, then I both satisfies and falsifies F. So the truth value 0.5 is said to be a meaningless point in fuzzy inference system [5].

<u>Definition 6 :</u> A formula is said to be unsatisfiable if and only if it is falsified by all its interpretations.

Bottom up inference

A bottom up refutation begins with assertions in the input set of clauses. It uses implications to derive new assertions from old ones, and ends with the derivation of assertions which explicitly contradict the denial of the goal. Bottom-up inference is a generalisation of instrantiation combined with the classical rule of modus ponens. Instantiation is restricted to the minimum needed to match assertions with conditions, so that modus ponens can be applied. A more precise definition of bottom up inference is given in [3]. Bottom up inference is a special case of the hyper resolution rule defined and proved by Robinson [2].

Bottom-up inference with fuzzy reasoning

The classifical bottom-up inference deals with pricise facts and rules. For further clarity interested readers are refered to [3].

Example 1 : In this case facts and rules are represented by fuzzy linguistic variables. But the left hand sides of the rules perfectly match with the given facts.

F1 : K is big => big (K) $\equiv \{k\} = \frac{8}{.7}, \frac{9}{.8}, \frac{10}{1}$

F2 : L is small => small (L) $\equiv \{l\} = \frac{1}{1}, \frac{2}{.5}, \frac{3}{.3}$

R1 : big (K) -> medium (M), where

$$\text{medium (M)} \equiv \{m\} = \frac{5}{.5}, \frac{6}{1}, \frac{7}{.5}$$

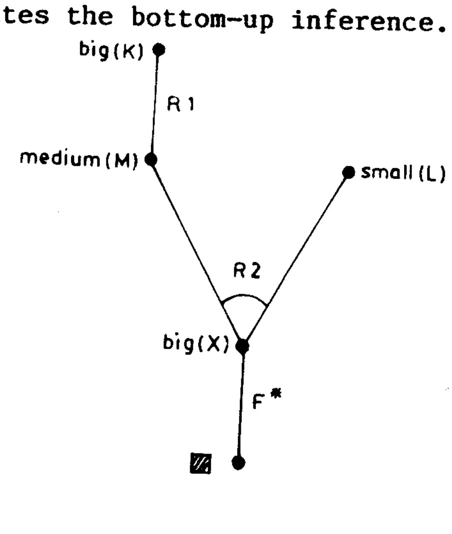
R2 : medium (M) and small (L) -> big(X), where

$$\text{big(X)} \equiv \{x\} = \frac{7}{.7}, \frac{8}{.8}, \frac{9}{.9}, \frac{10}{1}$$

Goal : X is big $\equiv$ big(X)

F* : Denial of the goal : $\sim$ big(X)

The figure below illustrates the bottom-up inference.



In this case we have applied Mamdani's translating rule R_c. It is seen that the deduction starts with the given assertions and derives (using the compositional rules of inference) the goal state "big(X)", which fuzzily contradicts the negation of the goal state, i.e., " $\sim$big(X)". The fuzzy contradiction is achieved by taking the intersection between "big(X)" and "$\sim$ big(X)". At the end of fuzzy contradiction we do not end up with the emply clause $\square$. Instead we achieve one formula represented by ▨ having some truth values. In this particular example we get, ▨ $\equiv$ (.3,.2,.1,0). To give an interpretation I to the formula ▨ we pick up the maximum membership value/values of the formula ▨ . If the maximum membership value/values is/are less than .5, then (according to the Definition 5) the formula is falsified. If the formula ▨ is falsified for the maximum membership value then it would be falsified for all other membership values. Thus the statement is unsatisfiable (according to the Definition 6). Hence the denial of the goal is inconsistent with the given facts and rules which are fuzzily stated.

<u>Example 2 :</u> In this case facts and rules are represented by fuzzy linguistic variables but the left hand sides of the rules do not perfectly match with given facts.

F1: K' is more or less big => more or less big (K') = $\{k'\}$

$$= \frac{7.5}{.6}, \frac{8}{.7}, \frac{9.5}{.9}$$

F2: L is small => small (L) $\equiv \{l\} = \frac{1}{1}, \frac{2}{.5}, \frac{3}{.3}$

R1 : big(K) - > Medium (M), where Medium(M) $\equiv \{m\} = \frac{5}{.5}, \frac{6}{1}, \frac{7}{.5}$

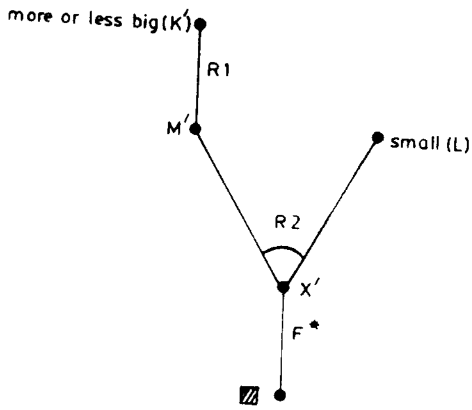
R2 : Medium (M) and small (L) - > big(X), where

$$big(X) \equiv \{x\} = \frac{7}{.7}, \frac{8}{.8}, \frac{9}{.9}, \frac{10}{1}$$

Goal : X is big $\equiv$ big(X)

F* : Denial of the goal : $\sim$ big(X)

The figure below illustrates the bottom-up inference.



Here M' = K'o R(k;M) = (.5, .9, .5)

and either X' (according to Zadeh's arithmetic rule)

$$\equiv \{x'\} = (M'oR(M;X)) \vee (LoR(L;X))$$

$$= (.7, .8, .9, 1)$$

or X' (according to the Mamdani's translating rule R_c)

$$\equiv \{x'\} = (M'oR(M;X)) \wedge (LoR(L;X))$$

$$= (.7, .8, .9, .9)$$

In this particular example we have to apply generalised modus ponens instead of modus pones. Hence we may either use

Zadeh's arithmatic rule R_a or Mamdani's translating rule R_c. Here the extended Hausdorff distance measures between the fuzzy sets of (K, K') and (X; X') are within the reasonable limit [4]. Ultimately we derive X' which is either having the truth values (.7, .8, .9, 1) or (.7, .8, .9, .9). When X' fuzzily contradicts F* we end up with the formula ▨ having truthvalues (.3, .2, .1, 0). Hence as per example 1 we can say that the denial of the goal is inconsistent with the given facts and rules which are fuzzily stated.

If the extended Hausdorff distance between any two matchable fuzzy sets is beyond the reasonable limit we can not say anything specifically about our inference mechanism.

In order to change a predicate into a proposition, each individual variable of the predicate must be bound. This may be achieved in two ways. One way is to bind an individual variable by assigning a value to it. The other method of binding individual variable is by quantification of the variable. The most common forms of quantification are universal and existential. By the approaches of substitution and unification of binary logic [5] we can easily extend the fuzzy resolution principle into first order logic.

<u>Example 3 :</u>

F1 : Mr. A is tall => tall (Mr.A) $\underline{=} \{h\} = \frac{5'9''}{.7}, \frac{6'}{.8}, \frac{6'5''}{1}$

F2 : Mr.A has average weight => average weitht (Mr.A) $\equiv \{w\}$

$$= \frac{60kg}{.7}, \frac{65kg}{.75}, \frac{70kg}{1}$$

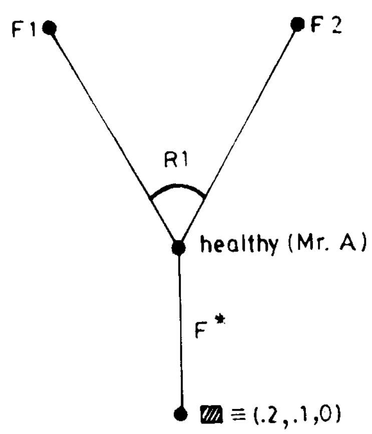
R1 : tall (Mr.X) and average weight (Mr.X) -> healthy(X),

where healthy(X) $\equiv \{x\} = \frac{(5'.9";65kg)}{.8}, \frac{(6'.3",70kg)}{.9}, \frac{(6'.5",75kg)}{1}$

Goal : Mr. A is healthy $\equiv$ healthy (Mr.A)

F* : Denial of the goal : $\sim$ healthy (Mr.A).

The figure below illustrates the bottom up inference.



Now we stipulate the following result.

<u>Theorem 1 :</u> (completeness of the resolution principle) : A set S of fuzzy formulas is unsatisfiable if and only if there is a deduction of the formula ▨ having maximum membership function below .5.

<u>Proof :</u> Deduction starts with the input assertions and derives (using the compositional rules of inference) the goal state which fuzzily contradicts the negation of the goal state.

Let all the possible membership functions of the goal state is either above of below the meaningless point .5; then the fuzzy contradictions which is the intersections between goal state and its negation wil always yield the formula ▨ having maximum membership function below .5. Hence according to the definitions 5 and 6 , S is unsatisfiable.

Conversely, let all the possible membership functions of the goal state is distributed on and/or around the meaningless point .5; then the fuzzy contradictions may always yield the formula ▨ having maximum membership function .5. If the maximum membership function is .5, it becomes a meaningless point for refutational inference.

Q.E.D.

By theorem 1 we understand that if the liguistic variable of the goal state is not very much fuzzy in naturte (i.e. the distribution of the membership functions is below or above the meaningless point .5) then the fuzzy resolution principle is complete. For the distribution of the membership function we may consider the S or π - function as proposed by several researchers.

Conclusion

By the fuzzy resolution principle we can perform inference in fuzzy propositional and first order logic as effectively as in binary logic. We have proved the completeness of the fuzzy resolution principle. The present approach is based on linguistic variable and compositional rules of inference. Based on the present fuzzy resolution principle we can have a more flexible logic programming system for uncertain environment.

References

(1) J.A. Robinson, A machine oriented logic based on the resolution principle, J.ACM, 12, 1, pp.23-41, 1965.

(2) ________________ , Automatic deduction with hyper-resolution, Int. J. Computer Math, 1, pp.227-234, 1965.

(3) ________________ , R. Kowalski, Logic for problem solving, North-Holland, 1979.

(4) D. Duois and H. Prade, On distance between fuzzy points and their use for plausible reasoing. Proc. IEEE Int.Conf. on Cybernetics and Society, Bombay - New Delhi, pp.300-303, Dec.30, 1983 - Jan.7, 1984.

(5) M. Mukaidono, Fuzzy inference of reasoning style, in : R.R. Yager Eds. Fuzzy sets and possibility theory, Pergamon Press, pp.224-231, 1982.

(6) R.C.T. Lee, Fuzzy logic and the resolution principle, J.ACM, 19, 1, pp.109-119, 1972.