INITIAL INQUISITION OF GENERALIZED TOPOLOGICAL SPACE

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ABSTRACT: In this paper the definition and their concepts of generalized topological space are introduced. And on this basis, the relationship between generalized topological space and grey(complex fuzzy) and fuzzy and general topological space is discussed. there are also studies of generalized continuous mapping and compactness of generalized topological space.

KEYWORDS: Generalized subset, generalized point, generalized topological space and generalized continuous mapping.

I. INTRODUCTION

We defined the grey topological space in [2]. Mr. Wu Heqin and wang Qingyin gave the conception and properties of generalized subsets. We shall study the generalized topological space.

Definition 1: Let X be a discussible field. If subordinate function of generalized subset A in X is equal to R(real number set) always, or $\mu_A(x)=R, \forall x \in X$, then A is called the whole generalized set in X, usually written X.

If subordinate function of generalized subset A in X is equal to{0}always, or $\mu_A(x)=\{0\}, \forall x \in X$, then A is called the empty generalized set in X, usually written E.

Definition 2: Let X be a discussible field. The generalized subset in X

$$\mu(x)=\begin{cases} M, & x=a \\ \{0\}, & x\neq a \end{cases}$$

where $M \in T$(Cantor set composed of all nonempty subset).

These are called the generalized point of X, written a_M or simplified a_M to a.

Definition 3: Let A,B be generalized subsets of X. If $\mu_A(a) \neq \{0\}$ and $\mu_B(a) \neq \{0\}$, then A and B are called the joint to point a.

Definition 4: Let A,B be generalized subsets of X. If $\inf \mu_A(a) < \sup \mu_{B^c}(a)$ or $\inf \mu_B(a) < \sup \mu_{A^c}(a)$, then A and B are called coincide to a.

A and B are called joint(coincide) if and only if there exists $a \in X$ such that A and B joint(coincide) to a.

Definition 5: Let A be a generalized subset of X, a_M be a generalized point of X.

(1) If $\sup M \leqslant \inf \mu_A(a)$, then a_M is called belong to A, usually written $a_M \in A$.

(2) If $\sup M > \inf \mu_{A^c}(a)$, then a_M is called coincide to A, usually written $a_M \triangle A$.

Definition 6: Let f be a mapping from set X to Y, B be a generalized subset in Y and subordinate function of B is $\mu_B(y), y \in Y$. By the inverse image of B(written $f^{-1}[B]$) we mean generalized subset in X and its subordinate function is defined by: $\mu_{f^{-1}[B]}(x) = \mu_B(f(x)), \forall x \in X$.

Let A be a generalized subset in X and subordinate function of A is $\mu_A(x), x \in X$. By the image of A(written $f[A]$), we mean generalized subset in Y and its subordinate function is defined by: $\forall y \in Y$,

$$\mu_{f[A]}(y) = \begin{cases} \bigcup\limits_{x \in f^{-1}(y)} \{\mu_A(x)\}, & f^{-1}(y) \neq \emptyset \text{(empty Cantor set)} \\ \{0\}, & f^{-1}(y) = \emptyset \end{cases}$$

where $f^{-1}(y) = \{x \mid f(x) = y\}$.

Theorem 1: Let f be a mapping from set X to Y, B is a generalized subset in Y, then

(1) $f^{-1}[B^C] = (f^{-1}[B])^C$.

(2) If f is a surjetion, then $f[f^{-1}[B]] = B$.

(3) And g is a mapping from Y to set Z, then there exists any generalized subset C of Z: $(g \cdot f)^{-1}[C] = f^{-1}[g^{-1}[C]]$,

where $g\cdot f$ is the compound mapping of f and g.

Proof: (1) Since $\mu_{f^{-1}[B^c]}(x)=\mu_{B^c}(f(x))=\{1-e \mid e\in \mu_B(f(x))\}$ $=\{1-e \mid e\in \mu_{f^{-1}[B]}(x)\}=\mu_{(f^{-1}[B])^c}(x), \forall x\in X$.

Hence $f^{-1}[B^c]=(f^{-1}[B])^c$.

(2) Since f is a surjetion, hence $f^{-1}(y)\neq\emptyset$. Thus $\mu_{f[f^{-1}[B]]}(y)$ $=\bigcup_{x\in f^{-1}(y)}\{\mu_{f^{-1}[B]}(x)\}=\bigcup_{x\in f^{-1}(y)}\{\mu_B(f(x))\}=\mu_B(y), \forall y\in Y$.

So $f[f^{-1}[B]]=B$.

(3) For $\forall x\in X$, $\mu_{(g\cdot f)^{-1}[C]}(x)=\mu_C(g\cdot f)(x))=\mu_C(g(f(x)))$ $=\mu_{g^{-1}[C]}(f(x))=\mu_{f^{-1}[g^{-1}[C]]}(x)$.

Hence $(g\cdot f)^{-1}[C]=f^{-1}[g^{-1}[C]]$.

II. CONCEPTION OF GENERALIZED TOPOLOGICAL SPACE

Definition 7: If generalized subset family $\mathfrak{J}$ in X satisfies:

(1) $X, E\in\mathfrak{J}$,

(2) If $A, B\in\mathfrak{J}$, then $A\cap B\in\mathfrak{J}$,

(3) If $A_t(t\in T)\in\mathfrak{J}$, then $\bigcup_{t\in T} A_t\in\mathfrak{J}$.

Then $\mathfrak{J}$ is called a generalized topology and $(X,\mathfrak{J})$ is called a generalized topological space. The elements A in $\mathfrak{J}$ are called generalized open subsets and A^C are called generalized closed subsets, where $\mu_{A^c}(x)=\{1-e \mid e\in \mu_A(x)\}, \forall x\in X$.

Definition 8: Let $(X,\mathfrak{J})$ be a generalized topological space. a_M is a genertlized point and A is a generalized subset in X. If there exists a generalized subset $B\in\mathfrak{J}$ such that $a_M\in B\subseteq A$, then A is called neighbourhood of a_M. If there exists a generalized subset $B\in\mathfrak{J}$ such that $a_M \triangle B\subseteq A$, then A is called a coincidence field of a_M.

We use $\mathcal{U}_{a_M}$ to express neighbourhood(or coincidence field) train composed of all neighbourhood(or coincidence field) of a_M.

Theorem 2: Let $(X,\mathfrak{J})$ be a generalized topological space, then neighbourhood(or coincidence field)train of the generalized point a of X has the following properties:

(1) $\mathcal{U}_a\neq\emptyset$.

(2) If $A\in\mathcal{U}_a$, then $a\in A$.

(3) If $A, B \in \mathcal{U}_a$, then $A \cap B \in \mathcal{U}_a$.

(4) If $B \in \mathcal{U}_a$ and a generalized subset $A \supseteq B$, then $A \in \mathcal{U}_a$.

Prof: We only prove neighbourhood.

(1) It is obvious $X \in \mathcal{U}_a$, hence $\mathcal{U}_a \neq \emptyset$.

(2) If $A \in \mathcal{U}_a$, then there exists $B \in \mathfrak{J}$ such that $a \in B \subseteq A$, hence $a \in A$.

(3) If $A, B \in \mathcal{U}_a$, then there exists $A_1, B_1 \in \mathfrak{J}$ such that $a \in A_1 \subseteq A$ and $a \in B_1 \subseteq B$, hence there exists $A_1 \cap B_1 \in \mathfrak{J}$ such that $a \in A_1 \cap B_1 \subseteq A \cap B$. So $A \cap B \in \mathcal{U}_a$.

(4) If $B \in \mathcal{U}_a$, then there exists $B_1 \in \mathfrak{J}$ such that $a \in B_1 \subseteq B$, hence $a \in B_1 \subseteq A$. So $A \in \mathcal{U}_a$.

Definition 9: Let $(X, \mathfrak{J})$ be a generalized topological space.

(1) $\bigcup_{a \in A} \{B \mid B \in \mathfrak{J}, a \in B \subseteq A\}$ is called the interior of A, usually written $\mathring{A}$.

(2) $\bigcap_{a \in A} \{C \mid C^c \in \mathfrak{J}, a \in A \subseteq C\}$ is called the cloure of A, usually written $\bar{A}$.

Theorem 3: (1) $\mathring{A}$ is the largest generalized open subset contained in A.

(2) $\bar{A}$ is the smallest generalized closed subset containing A.

Theorem 4: The generalize subset A is the generalized open (or closed)set if and only if $A = \mathring{A}$(or $\bar{A}$).

Theorem 5: Let $(X, \mathfrak{J})$ be a generalized topological space. If the generalized point $a_M \in \bar{A}$, then every coincidence field of a_M coincide to A all.

Proof: $a_M \in \bar{A} \rightarrow$ Any generalized closed subsets $B \supseteq A$, there is always $a_M \in B$, i.e. $\inf \mu_B(a) \geqslant \sup M$.

$\rightarrow$ Any generalized open subsets $C \subseteq A^c$, always holding:

$\inf \mu_C(a) \leqslant \sup \mu_C(a) \leqslant \inf \mu_{A^c}(a) = \inf\{1 - e \mid e \in \mu_A(a)\} \leqslant \sup M$, or $\sup M < \inf \mu_{C^c}(a)$.

$\rightarrow$ Any generalized open subsets C satisfies $\sup M > \inf \mu_{C^c}(a)$ did not contain in A, so C and $(A^c)^c = A$ are coincide.

$\rightarrow$ Any generalized open coincidence field C of a_M always coincide with A.

→Every coincidence field of a_M always coincide with A.

III. RELATIONSHIP BETWEEN GENERALIZED TOPOLOGICAL SPACE AND GREY(COMPLEX FUZZY) AND FUZZY AND GENERAL TOPOLOGICAL SPACE

1. Let $(X, \mathfrak{J})$ be a generaized topological space. If for any generalized subsets A in X, $\mu_A(x) \subseteq [0,1], \forall x \in X$. Then change the generalized subsets A into the grey(complex fuzzy)subsets. Hence change $\mathfrak{J}$ into the grey(complex fuzzy)topology and change $(X, \mathfrak{J})$ into the grey(complex fuzzy)topological space.
2. Let $(X, \mathfrak{J})$ be a grey(complex fuzzy)topological space. If the upper and lower subordinate functions of the grey(complex fuzzy)subsets A are erual, or $\overline{\mu}_A(x) = \underline{\mu}_A(x), \forall x \in X$. Then change A into the fuzzy subsets. Hence change $\mathfrak{J}$ into the fuzzy topology and change $(X, \mathfrak{J})$ into the fuzzy topological space.
3. Let $(X, \mathfrak{J})$ be a fuzzy topological space. If subordinate functions of the fuzzy subsets A, $\mu_A(x) \in \{0,1\}, \forall x \in X$. Then change A into the Cantor subsets. Hence change $\mathfrak{J}$ into the general topology and change $(X, \mathfrak{J})$ into the general topological space.

Hence general topological space is a particular example of fuzzy topological space. Fuzzy topological space is a particular example of grey(complex fuzzy)topological space. Grey(complex fuzzy)topological space is a particular example of generalized topological space.

So generalized topological space $\supsetneqq$ grey(complex fuzzy) topological space $\supsetneqq$ fuzzy topological space $\supsetneqq$ general topological space.

IV. GENERALIZED CONTINUOUS MAPPING AND COMPACTNESS OF GENERALIZED TOPOLOGICAL SPACE

Definition 10: The mapping from the generalized topological space $(X_1, \mathfrak{J}_1)$ to the generalized topological space $(X_2, \mathfrak{J}_2)$ is called generalized continuous if and only if $\forall B \in \mathfrak{J}_2 \rightarrow$ $\rightarrow f^{-1}[B] \in \mathfrak{J}_1$.

Theorem 6: The composition mapping of two generalzed contiuous mappings is also the generalized continuous mapping.

Proof: Let generalized continuous mapping $f: (X_1,\mathfrak{J}_1)\to(X_2,\mathfrak{J}_2)$, $g: (X_2,\mathfrak{J}_2)\to(X_3,\mathfrak{J}_3)$. If $\forall C\in\mathfrak{J}_3$, then $(g\cdot f)^{-1}[C]=f^{-1}[g^{-1}[C]]$.

Since g is generalized continuous, hence $g^{-1}[C]\in\mathfrak{J}_2$.

Since f is generalized continuous, hence $f^{-1}[g^{-1}[C]]\in\mathfrak{J}_1$.

So $(g\cdot f)^{-1}[C]\in\mathfrak{J}_1$. Hence $g\cdot f$ is generalized continuous.

Theorem 7: The mapping from the genralized topological space $(X_1,\mathfrak{J}_1)$ to the generalized topological space $(X_2,\mathfrak{J}_2)$ is generalized continuous if and only if $\forall B^C\in\mathfrak{J}_2\to f^{-1}[B^C]\in\mathfrak{J}_1$.

Definiion 11: The gneralized subsets family $\{At\mid t\in T\}$ is called the cover of the generalized subset B if and only if $B\subseteq\bigcup At$. If At are all generalized open subaets, this is called open cover. If some subfamily is still coverd, then this is called subcover.

Definition 12: The generalized topological space $(X,\mathfrak{J})$ is called compact if and only if every open cover has finite subcover.

Theorem 7: Let f be a generalized continuous surjetion from the generalized topologial space $(X_1,\mathfrak{J}_1)$ to the generalized topological space $(X_2,\mathfrak{J}_2)$. If $(X_1,\mathfrak{J}_1)$ is compact, then $(X_2,\mathfrak{J}_2)$ is compact too.

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