GREY FUNCTIONS --- COMPLEX FUZZY FUNCTIONS

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The concept of function is one of the important basic concepts in the classical mathematics. Now, we would like to set up the concept of grey functions (or complex fuzzy functions) in the rational grey numbers (or complex fuzzy numbers) to make preparation for the study of the grey limits and grey derivatives and for the further establishment of grey calculus.

I. Concept of the Grey Functions

From Reference (2)*, it is known that both the interval-type and the information-type grey numbers are called, by a joint name, the rational grey numbers. But the interval-type grey number is a kind of special fuzzy numbers. Thus, the rational grey number is also called the complex fuzzy number.

We define the grey functions (or complex fuzzy functions) as follows:
Definition 1. Suppose C denotes the set of rational grey numbers, and D is the subset of C, i.e. $D \subseteq C$. If

$$f \subseteq D \times C = \{(x,y) \mid x \in D,\ y \in C\}$$

and the following conditions are met, they are, for any arbitrary $(x_1, y_1) \in f$, $(x_2, y_2) \in f$, and $x_1 = x_2$, there must be $y_1 = y_2$, then, the f is called the grey function (or complex fuzzy function) defined at D and is expressed as $y = f(x)$, $x \in D$.

Obviously, when C is the fuzzy set, and D is the set of the interval-type grey numbers, the grey function is fuzzy function.

Example 1. If $D = \{[1,2],1\}$, then

$$f = \{(1,[1,1]),\ ([1,2],1)\}$$

is a grey function, whose domain of definition is D, and

$$f(1) = [\underset{\sim}{1},1], \qquad f([\underset{\sim}{1},2]) = 1$$

We point out specially: As the rational grey numbers contain all of the real numbers, there may exist the fact that the x and y in (x,y), the element of f, are both real numbers, Then, f is the real function in classical calculus.

Definition 2. For arbitrary grey function $y = f(x)$, $x \in D$, when $D = \{1,2,3,\ldots,n,\ldots\}$, i.e. D is a set of natural numbers, $y = f(x)$ is called a sequence of grey numbers, or simply called grey sequence. The grey sequence can be also expressed as:

$$f(1),\ f(2),\ \ldots,\ f(n),\ldots \text{ or}$$

$$\text{simplified as } f_1,\ f_2,\ \ldots,\ f_n,\ldots,$$

If f_n is denoted by a_n, then, $a_1, a_2, \ldots, a_n$ is called a sequence of grey numbers, and a_n is called the general term of the grey sequence. Here a_n is defined in the set of classical rational grey numbers, and $a_n = \mu(x_n) = f_n$. When a_n, $n = 1,2,3,\ldots n$, are all the real numbers, the sequence of grey numbers $a_1, a_2, \ldots, a_n, \ldots$ is the sequence of real numbers in classical mathematics.

II. Grey Extension of the Real Functions

The rational grey numbers are the extension of real numbers, and the grey functions are the extension of real functions as well. A variety of real functions especially those very useful elementary functions are studied in the classical calculus. Here we want to give a rule for making the real functions to realize the grey extension and under this rule, all real functions can be extended to the grey functions whose domain of definitions and the value ranges are all the rational grey numbers.

Definition 3. If $y = f(x)$ is a real function, and both x and y take only real values, then the grey extension stipulates:

$$f([a,b]) = \left[\begin{matrix}\inf f(x), & \sup f(x) \\ x\in[a,b], & x\in[a,b]\end{matrix}\right]$$

$$f([\underset{\sim}{a},b]) = \left[\begin{matrix}\inf f(x), & \sup f(x) \\ x\in[a,b], & \underset{\sim}{\ } x\in[a,b]\end{matrix}\right]$$

in which, f(x) is tenable only when there exist at least one point x_0 in [a,b], i.e. $x_0 \in [a,b]$, and $f(x_0)$ is meaningful. Otherwise, $f([a,b])$ and $f([\underset{\sim}{a},b])$ are considered meaningless.

In this way, we extend the domain of definition of $y = f(x)$ to rational grey numbers. The obtained grey function is still denoted as $y = f(x)$, and it will be still the original real function only when the x being discussed is the real number.

Example 2. Extend $y = \sin x$ to a grey function, $x \in [a,b]$.

Solution:

$$\sin[a,b] = \left[\inf_{x\in[a,b]} \sin x, \quad \sup_{x\in[a,b]} \sin x\right]$$

$$\sin[\underset{\sim}{a,b}] = \left[\inf_{x\in[a,b]} \sin x \underset{\sim}{,} \quad \sup_{x\in[a,b]} \sin x\right]$$

We have rational grey function $y = \sin x$, and call it the grey sinusoidal function.

Definition 4. If a real function $y = f(x)$ is an elementary function, then the grey function obtained through grey extension will be called grey elementary function, still expressed as $y = f(x)$.

Example 3. Find the grey elementary function of $y = x$ (x is a real number).

Solution:

$$f([a,b]) = \left[\inf_{x\in[a,b]} x, \quad \sup_{x\in[a,b]} x\right] = [a,b]$$

$$f([\underset{\sim}{a,b}]) = \left[\inf_{x\in[a,b]} x \underset{\sim}{,} \quad \sup_{x\in[a,b]} x\right] = [\underset{\sim}{a,b}]$$

Then, the grey elementary function is $y = x$. Here x is an arbitrary rational grey number.

Example 4. Find the grey function of $y = e^x$, (x is a real number).

Solution:

$$f([a,b]) = \left[\inf_{x\in[a,b]} e^x, \quad \sup_{x\in[a,b]} e^x\right]$$

$$f([\underset{\sim}{a,b}]) = \left[\inf_{x\in[a,b]} e^x \underset{\sim}{,} \quad \sup_{x\in[a,b]} e^x\right]$$

Then, $y = e^x$ is a grey function. Here x is an arbitrary rational grey number, thus this function is called grey exponential function.

Example 5. Find the grey function of $y = \ln x$ (x is a real number, $x > 0$).

Solution: $f([a,b]) = \begin{bmatrix} \inf\limits_{x \in [a,b]} \ln x, & \sup\limits_{x \in [a,b]} \ln x \end{bmatrix} \quad 0 < a < b$

$$f([\tilde{a,b}]) = \begin{bmatrix} \inf\limits_{x \in [a,b]} \ln x, & \sim \sup\limits_{x \in [a,b]} \ln x \end{bmatrix} \quad 0 < a < b$$

Then, $y = \ln x$ is the grey function. Here x is an arbitrary classical rational grey number, thus this grey function is called grey logarithmic function.

III. Grey Extension Derivatives

Definition 5. Suppose $y = f(x)$ is an arbitrary real function in the calculus in classical mathematics, when $f'(x)$ exists, its grey extension function $f'(x)$ is then called the extension grey derivative of the grey extension function of $f(x)$.

Example 6. Find the extension grey derivative of $y = e^x$.

Solution: $\because y' = e^x$, $\therefore$ the grey extension derivative of $y = e^x$ is $y' = e^x$, i.e. the extension grey derivative of the grey extension function of $y = e^x$ is $y = e^x$.

* References

1. Deng Julong: Grey Control System (in Chinese), the Central China University of Science and Technology Press, 1985.

2. Wu Heqin and Yue Changan: Introduction on Grey Mathematics (in Chinese), Hebei People's Education Publishing House, 1989.