A NEW KIND OF FUZZY SET AND ITS APPLICATIONS TO UNCERTAINTY ANALYSIS

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Abstract

In this paper, we have proposed a new kind of fuzzy set, which was named MW-fuzzy set, and described incompleteness of information and uncertainty of system by them. In addition, the basic operations and relations on the new fuzzy sets have been endowed with certain practical significance from angle of information theory.

Keywords: MW-fuzzy set, TC-transformation, ED-transformation, Grey information, Uncertainty analysis.

1. Introduction and symbols

As a generalization of interval-valued fuzzy sets [3], Meng [1] has presented the concept of interval-value L-fuzzy set, and established many decomposition theorems for it. In order to deal with the incompleteness of information and the uncertainty of system, we have put forward in this paper a kind of fuzzy set called MW-fuzzy set based on interval-valued fuzzy sets.

Information is a fundamental concept in system science, it mirrors the important characteristics of system. In general, informations fall roughly into three categories: completely known, incompletely known and completely unknown, they are called white, grey and black information respectively. For example, we have got some informations about John as follows:

Age: 30; Height: 1.68 1.72 m; Weight: 60 70 kg; Acadmic degree:

unknown;Nationality:China or Japan or Korea.

Among this informations, age is white, acadmic degree is black, height and weight and nationality are grey.

Meng [2] has ever described black and white informations from point of view of set, but missed out a lot of grey informations. In this paper, we try to express the incompleteness of informations and the uncertainty of systems by MW-fuzzy sets.

Throughout this paper we agree on

R: real set, N: natural set, I: real unit interval [0,1],

$[I]=\{[a,1]: 0 \leqslant a \leqslant 1, a \in R\}$, $[R]=\{[a,b]: a \leqslant b, a,b \in R\}$,

$M=\{(\frac{1}{n}, \frac{2}{n}, \cdots, \frac{n-1}{n}, 1): n \in N\}$, $a \vee b=\max(a,b)$, $a \wedge b=\min(a,b)$.

2. MW-fuzzy sets

Definition 2.1. Let X be a usual set. The mapping

$$A: X \to [I] \cup M$$

is called a MW-fuzzy set on X. All MW-fuzzy sets on X is denoted by G(X).

Remark 2.1. If for every $x \in X$, $A(x) \in [I]$, then A just is an interval-valued fuzzy set on X, therefore MW-fuzzy set is a generalization of interval-valued fuzzy set.

Definition 2.2. Let $A, B \in G(X)$ and $x \in X$.

(1) If $A(x)=[a(x),1]$, $B(x)=[b(x),1]$, then

$A(x) \leqslant B(x)$ iff $a(x) \leqslant b(x)$; $A(x)=B(x)$ iff $a(x)=b(x)$;

$A(x) \vee B(x)=[a(x) \vee b(x), 1]$; $A(x) \wedge B(x)=[a(x) \wedge b(x), 1]$.

(2) If $A(x)=(\frac{1}{m}, \frac{2}{m}, \cdots, \frac{m-1}{m}, 1)$,

$B(x)=(\frac{1}{n}, \frac{2}{n}, \cdots, \frac{n-1}{n}, 1)$, then

$A(x) \leqslant B(x)$ iff $m \geqslant n$; $A(x)=B(x)$ iff $m=n$;

$A(x) \vee B(x)=(\frac{1}{m \wedge n}, \frac{2}{m \wedge n}, \cdots, 1)$; $A(x) \wedge B(x)=(\frac{1}{m \vee n}, \frac{2}{m \vee n}, \cdots, 1)$.

(3) If $A(x)=[a(x),1]$ and $a(x)\neq 1$, $B(x)=(\frac{1}{n},\frac{2}{n},\cdots,1)$, then $A(x)\leqslant B(x)$ and $A(x)\neq B(x)$;

$$A(x)\vee B(x)=(\frac{1}{n},\frac{2}{n},\cdots,1);\quad A(x)\wedge B(x)=[a(x),1].$$

Definition 2.3. Let $A,B\in G(X)$, then

(1) $A\subset B$ iff $A(x)\leqslant B(x)$ for any $x\in X$;

(2) $A=B$ iff $A(x)=B(x)$ for any $x\in X$;

(3) $(A\cup B)(x)=A(x)\vee B(x)$;

(4) $(A\cap B)(x)=A(x)\wedge B(x)$.

The following two theorems follow easily from Definition 2.2 and 2.3 .

Theorem 2.1. $(G(X),\subset)$ is an ordered set with a maximal 1_X and a minimal element 0_X, where

$$1_X(x)\equiv 1,\ x\in X,$$
$$0_X(x)\equiv[0,1],\ x\in X.$$

Theorem 2.2. $(G(X),\subset,\cup,\cap)$ is a lattice.

3. TC-transformation

In order to describe the uncertainty of systems, we need some technical preparations. It is in order to express the incompleteness of a kind of grey information that TC-transformation was introduced.

Definition 3.1. Let $[a,b]\in[R]$, the mapping

$$T:[a,b]\to I\ ,\qquad x\mapsto T(x)=\frac{1+x-a}{1+b-a},$$

is called translation-compression transformation, or TC-transformation for short. $T[a,b]$ is called the image of $[a,b]$ under T.

It is easily seen that $T[a,b]=[\frac{1}{1+b-a},1]$, and $T[a,b]\to[0,1]$ when $b-a\to\infty$.

In order to state the properties of TC-transformation we give the following:

Definition 3.2. On [R] define an order $\prec$ as follows:

$$[a_1, b_1] \prec [a_2, b_2] \text{ iff } b_1 - a_1 \leqslant b_2 - a_2,$$

we call $\prec$ an interval-order.

Clearly $([R], \prec)$ is a linear order set.

The following theorem gives basic properties of TC-transformation.

Theorem 3.1. Let T be a TC-transformation, then

(1) T is order-preserving, that is, if $x_1 \leqslant x_2$ then $T(x_1) \leqslant T(x_2)$.

(2) T is interval-order-preserving, that is, if $[a_1, b_1] \prec [a_2, b_2]$ then $T[a_1, b_1] \prec T[a_2, b_2]$.

Proof. (1) It follows from T(x) is strictly increasing function.

(2) From (1) we get

$$T[a_i, b_i] = [T(a_i), T(b_i)] = [\frac{1}{1+b_i-a_i}, 1],$$

$$T(b_i) - T(a_i) = \frac{b_i - a_i}{1+b_i-a_i}, \quad i=1,2.$$

But function

$$f(x) = \frac{x}{1+x} \quad (x \geqslant 0)$$

is strictly increasing, hence

$$T(b_1) - T(a_1) \leqslant T(b_2) - T(a_2) \text{ when } b_1 - a_1 \leqslant b_2 - a_2.$$

This shows

$$T[a_1, b_1] \prec T[a_2, b_2] \text{ when } [a_1, b_1] \prec [a_2, b_2].$$

Remark 3.1. (1) TC-transformation is a one-to-one mapping between [a, b] and $[\frac{1}{1+b-a}, 1]$.

(2) TC-transformation can change closed intervals which lengths are equal into same closed interval, for example, [8, 12] and [100, 104] are transformed into $[\frac{1}{5}, 1]$.

4. ED-transformation

In order to express the incompleteness of another kind of grey information, we now introduce ED-transformation.

Definition 4.1. Given m ($m \in N$) numbers

$$r_1, r_2, \cdots, r_m, \qquad (4-1)$$

the mapping

$$p: r_i \longrightarrow \frac{i}{m}, \quad i=1,2, \quad ,m,$$

is called equidistribution transformation,or ED-transformation for short.

Clearly,sequence of numbers (4-1) is changed into that in [0,1] '

$$\frac{1}{m}, \frac{2}{m}, \cdots, \frac{m-1}{m}, 1,$$

this is m points which are well-distributed dispersed over [0,1].

Remark 4.1. The image $p(r_i) = \frac{i}{m}$ of r_i has nothing to do with concrete value of r_i, and has something to do with only lower mark i,thus sequence of numbers (4-1) can be seen sequence of points.In addition,different sequence of points probably corresponds to same one,for example,both $(a_1, a_2, \cdots, a_{10})$ and $(b_1, b_2, \cdots, b_{10})$ correspond to $(\frac{1}{10}, \frac{2}{10}, \cdots, 1)$.

5.Expression for uncertainty of systems

First we consider the concept of MW-fuzzy set from angle of information theory.

Suppose S is a system with uncertainty,that is,S contains three informations:white,gery and black information.All informations of S is denoted by usual set X.Now on X define a mapping

$$A: \quad X \rightarrow [I] \cup M$$

as follows:

(1) If $x \in X$ is a white information of S,then $A(x)=1 \triangleq [1,1]$.

(2) If $x \in X$ is a black information of S,then $A(x)=[0,1]$.

(3) If $x \in X$ is a grey information of S,then $A(x)=[a,1]$ or

$$A(x) = (\frac{1}{m}, \frac{2}{m}, \cdots, 1).$$

Clearly,the mapping A is a MW-fuzzy set on X,and the practial significance of A is that a description for uncer tainty of system S.

Remark 5.1. The more smaller (bigger) a in $A(x)=[a,1]$ is, or the more bigger(smaller) m in $A(x)=(\frac{1}{m}, \frac{2}{m}, \cdots, 1)$ is,then the more bigger (smaller) the uncertainty of information x is,and we say that the more greyyer (whiter) information x is.

Example 5.1.Now we consider the informations about John in section 1.Suppose that

$$X=(\text{age},\text{height},\text{acadmic degree},\text{nationality})$$
$$=(x_1, x_2, x_3, x_4, x_5).$$

Getting rid of dimension of height and weight,we obtain two closed intervals [1.68,1.72] and [60,70],then from TC-transformation,

$$T[1.68,1.72]=[\frac{1}{1.04}, 1], \qquad T[60,70]=[\frac{1}{11}, 1].$$

For nationality,we set up a sequence of point

$$(a_1, a_2, a_3)=(\text{China},\text{Japan},\text{Korea}),$$

by ED-transformation,

$$p(a_1, a_2, a_3)=(\frac{1}{3}, \frac{2}{3}, 1).$$

Let us regard John as a system,and denote its uncertainty by a MW-fuzzy set (see Fig.1)

$$A: X=(x_1, x_2, \cdots, x_5) \rightarrow [I] \cup M,$$
$$A(x_1)=1,$$
$$A(x_2)=[\frac{1}{1.04}, 1],$$
$$A(x_3)=[\frac{1}{11}, 1],$$
$$A(x_4)=[0, 1],$$
$$A(x_5)=(\frac{1}{3}, \frac{2}{3}, 1).$$

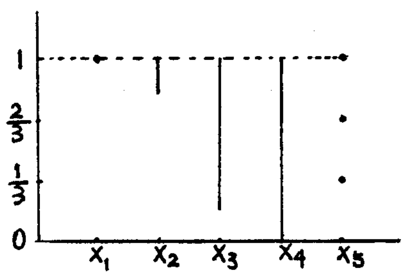

Fig.1

Remark 5.2.Considering from point of view of information theory,the maximal element 1_X of $(G(X), \subset)$ implies that all informations of system S are completely known,and the minimal element 0_X denotes that we known nothing about S.

The basic law knowing thing is from unknown to some extent understood and then fully known,that is,from black via grey to white.Now we portray the law by MW-fuzzy sets.

Let us consider geological structure of the moon.In the past mankind knew nothing about it,that is,it is a black information.With success of reaching the moon,it is transformed into grey information.Easily to imagine,with the development of science and technology, it will be changed gradually into white information. Now we denote the information by $x(t)$, where t is time, and let

$$X=\{x(t): t \text{ continuously change}\}.$$

On X define a MW-fuzzy set A (see Fig.2) as follows:

(1) $A(x(t))=[a(t),1]\in[I]$,

(2) $a(t)$ is increasing function about t,

(3) $a(t)\to 1$ when $t\to\infty$.

Clearly the MW-fuzzy set A vividly portraies this knowing process from black via grey to white.

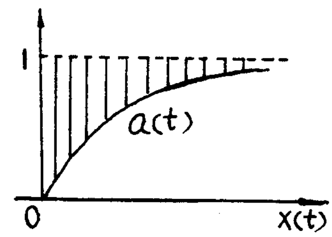

Fig.2

6. Practical significance of operations and relations on MW-fuzzy sets

For identical system S, at varying moment, it is, in general, different that our level of knowing informations of S, and for varying human, at same moment, it is incompletely same to know S. Describing these distinctness, we need a lot of MW-fuzzy sets, this is just concrete background of defining many MW-fuzzy sets on the same set X.

Suppose that X is the set of all informations about system S, and $A,B\in G(X)$. Let us consider significance of relation "$\subset$" from angle of information theory. We look upon A and B as Hans and John's levels of understanding about S respectively, then $A\subset B$ implies that Hans understood informations about S is not so clear as John. If we regard A and B as degree of grasping about S at moment t_1 and t_2 respectively, then $A\subset B$ shows that S was known at t_1 is not so penetrating as t_2 .

Nextly let us consider the significance about $A\cup B$. We yet look upon A and B as Hans and John's levels of understanding about S respectively, then $A\cup B$ implies that degree of grasping about S after Hans and John exchanged informations and learned from others' strong points to offest one's weaknesses. Further, $A\subset A\cup B$ and $B\subset A\cup B$ show that Hans or John's degree of grasping about S is not so clear as that of cooperating with each other.

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