Fuzzy Reliability
for Clear-event & Fuzzy-probability Mode

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Abstract : In this paper a preliminary research of the fuzzy reliability for clear-event & fuzzy-probability mode is proceeded. Some new concepts about fuzzy reliability, fuzzy failure rate, and fuzzy mean life are defined, and a series of related mathematical models are also established.

1. Introduction

In daily life, there is a certain kind of question, like this one, "Is this product reliable?" and to answer it, replying "Its reliability is 0.98" is certainly not more directly and understood than saying "It is very reliable", or "It is extremely reliable". Because in many situations what people concern is not the understanding to the accurate data but only the whole conception about a certain product, and it is more convenient to be expressed with fuzzy language. Then what proper fuzzy language should we choose? We call this kind of question clear-event & fuzzy-probability mode question, simply denoted by CF mode.

2. Definition of Fuzzy Reliability for CF Mode

We call reliability for CF mode fuzzy reliability, as it is named, a CF mode has a clear event and a fuzzy probability, and its answer should be described with a fuzzy linguistic value, not a accurate digit. So from the properties and their correlation between CF mode and general mode, we can extend the definition of reliability for general mode to fuzzy reliability for CF mode as follows: fuzzy reliability is the ability described with fuzzy linguistic value of a product to perform a required function under stated conditions for a stated period of time.

3. Indices of Fuzzy Reliability for CF Mode

For being simplified, only three main indices which are fuzzy reliability, fuzzy failure rate, and fuzzy mean life are extended from the theory of general reliability and studied in the sequel.

1)Fuzzy Reliability

We define that fuzzy reliability is the fuzzy linguistic probability of a product to perform a required function under stated conditions for a stated period of time,and denoted by $\underset{\sim}{R}(\omega)$ in which ω represents a clear event.

By definition,a fuzzy set representing linguistic values of probability is called a linguistic-valued fuzzy set,which is additionally divided into nine basic fuzzy subsets, that is, "extremely reliable","very reliable","reliable", "comparatively reliable","critically reliable","comparatively unreliable","unreliable","very unreliable",and "extremely unreliable", which are simply denoted by $\pi_1,\pi_2,\pi_3,\pi_4,\pi_5,\pi_6,\pi_7,\pi_8,$and π_9 in turn.Using these nine basic fuzzy subsets,we can character the linguistic-valued space of fuzzy reliability entirely. Then,if we want to know the fuzzy reliability for CF mode toward a certain product, we can describe it by using one of these fuzzy subsets. But note that these nine basic fuzzy subsets are exactly divided,that is,among them there is not any relation of inclusion.

Now we can give the mapping relation from the value of general reliability to the linguistic value of fuzzy reliability as follows:

$$\underset{\sim}{R}(\omega)=\pi_j, \qquad \text{for } \pi_j\in \mathcal{E}, \text{ and } j=1,\ldots,9,$$

in which

ω :a clear event,i.e."The value of general reliability is R",

$\mathcal{E}$:the linguistic-valued space of fuzzy reliability,

π_j :a basic fuzzy subset,or a linguistic value of fuzzy reliability.

The next thing we must resolved is determinating the membership functions of the nine basic fuzzy subsets. After studying the relevance between probability and the definition of reliability,we come to the conclusion that values of reliability have to be described through values of probability completely,or say, values of reliability are just what values of probability are, that is also to say, values of probability and corresponding values of reliability are equivalent, such as, "extremely probable" and "extremely reliable", "very probable" and "very reliable", and so on.

According to the theory of fuzzy probability, we know that the membership function of the fuzzy linguistic value, "very probable", is defined by

$$\mu(P \mid \text{very probable}) = \begin{cases} 0, & \text{for } 0 \leqslant P \leqslant a, \\ 2\left(\frac{P-a}{1-a}\right)^2, & \text{for } a \leqslant P \leqslant \frac{a+1}{2}, \\ 1-2\left(\frac{P-1}{1-a}\right)^2, & \text{for } \frac{a+1}{2} \leqslant P \leqslant 1, \end{cases}$$

in which a is a parameter limited by $\frac{1}{2}<a<1$.

Then, if we replace the value of probability P by the value of reliability R, of course we obtain the formula for the definition of the term "very reliable", and we temporarily add a symbol "*" on it, as the following

$$\mu(R \mid \text{very reliable}^{*}) = \begin{cases} 0, & \text{for } 0 \leqslant R \leqslant a, \\ 2\left(\frac{R-a}{1-a}\right)^2, & \text{for } a \leqslant R \leqslant \frac{a+1}{2}, \\ 1-2\left(\frac{R-1}{1-a}\right)^2, & \text{for } \frac{a+1}{2} \leqslant R \leqslant 1. \end{cases}$$

After using the definitions of the mood operators in fuzzy mathematics and the notations of $\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8$, and π_9, we have

$$\mu(R \mid \pi_1^{*}) = \left[\mu(R \mid \pi_2^{*})\right]^2, \qquad \mu(R \mid \pi_3^{*}) = \left[\mu(R \mid \pi_2^{*})\right]^{\frac{1}{2}},$$
$$\mu(R \mid \pi_4^{*}) = \left[\mu(R \mid \pi_3^{*})\right]^{3/4}, \qquad \mu(R \mid \pi_6^{*}) = \mu(1-R \mid \pi_4^{*}),$$
$$\mu(R \mid \pi_5^{*}) = \mu(R \mid \varepsilon) \wedge \left[\mu(R \mid \pi_4^{*})\right]^{c} \wedge \left[\mu(R \mid \pi_6^{*})\right]^{c},$$
$$\mu(R \mid \pi_7^{*}) = \mu(1-R \mid \pi_3^{*}), \qquad \mu(R \mid \pi_8^{*}) = \mu(1-R \mid \pi_2^{*}),$$
$$\mu(R \mid \pi_9^{*}) = \mu(1-R \mid \pi_1^{*}),$$

in which

$$\mu(R \mid \varepsilon) = 1, \qquad \text{for } 0 \leqslant R \leqslant 1,$$

then we can obtain all the membership functions of the nine basic fuzzy subsets, which are omitted in this paper.

Now it seems that we can easily judge in which degree a value of general reliability R ($\in [0,1]$) belongs to every basic fuzzy subset. But when we arbitrarily give a value of parameter a, of course limited by $\frac{1}{2}<a<1$, and draw the curved lines of the nine membership functions (see fig.1), we will find that there is relation of fuzzy inclusion among them, clearly, the relation can be represented by

$$\pi_1^* \subseteq \pi_2^* \subseteq \pi_3^* \subseteq \pi_4^*, \qquad \pi_9^* \subseteq \pi_8^* \subseteq \pi_7^* \subseteq \pi_6^*.$$

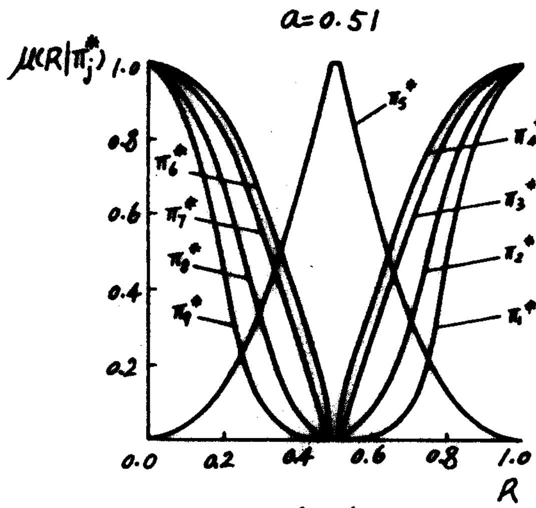

fig. 1

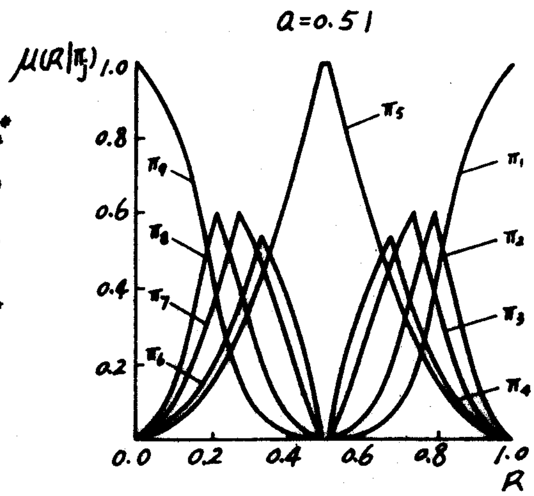

fig. 2

Certainly the conclusion above is contradictory to the notion of the linguistic-valued space of fuzzy reliability defined formerly, so the question now, is how to convert the functions with inclusion into the ones without any inclusion which describe the basic fuzzy subsets really.

The method is "digging", such as "digging" π_1^* out of π_2^*, and so on, then we have the following equations

$$\mu(R|\pi_2) = \mu(R|\pi_2^*) \wedge \mu(R|\pi_1^{*c}),$$
$$\mu(R|\pi_3) = \mu(R|\pi_3^*) \wedge \mu(R|\pi_2^{*c}),$$
$$\mu(R|\pi_4) = \mu(R|\pi_4^*) \wedge \mu(R|\pi_3^{*c}),$$

and some others

$$\mu(R|\pi_1) = \mu(R|\pi_1^*), \qquad \mu(R|\pi_5) = \mu(R|\pi_5^*),$$
$$\mu(R|\pi_6) = \mu(1-R|\pi_4), \qquad \mu(R|\pi_7) = \mu(1-R|\pi_3),$$
$$\mu(R|\pi_8) = \mu(1-R|\pi_2), \qquad \mu(R|\pi_9) = \mu(1-R|\pi_1),$$

so we can obtain the correct **membership** functions which are not with the symbol "*" from now on, and given by

$$\mu(R|\pi_1) = \begin{cases} 0, & \text{for } 0\leqslant R\leqslant a, \\ 4\left(\frac{R-a}{1-a}\right)^4, & \text{for } a\leqslant R\leqslant 0.5(a+1), \\ \left[1-2\left(\frac{R-1}{1-a}\right)^2\right]^2, & \text{for } 0.5(a+1)\leqslant R\leqslant 1, \end{cases}$$

$$\mu(R|\pi_2) = \begin{cases} 0, & \text{for } 0\leqslant R\leqslant a, \\ 2\left(\frac{R-a}{1-a}\right)^2, & \text{for } a\leqslant R\leqslant 0.5(a+1), \\ 1-2\left(\frac{R-1}{1-a}\right)^2, & \text{for } 0.5(a+1)\leqslant R\leqslant 1-0.4370(1-a), \\ 1-\left[1-2\left(\frac{R-1}{1-a}\right)^2\right]^2, & \text{for } 1-0.4370(1-a)\leqslant R\leqslant 1, \end{cases}$$

$$\mu(R|\pi_3) = \begin{cases} 0, & \text{for } 0\leqslant R\leqslant a, \\ \sqrt{2}\frac{R-a}{1-a}, & \text{for } a\leqslant R\leqslant a+0.4370(1-a), \\ 1-2\left(\frac{R-a}{1-a}\right)^2, & \text{for } a+0.4370(1-a)\leqslant R\leqslant 0.5(a+1), \\ 2\left(\frac{R-1}{1-a}\right)^2, & \text{for } 0.5(a+1)\leqslant R\leqslant 1, \end{cases}$$

$$\mu(R|\pi_4) = \begin{cases} 0, & \text{for } 0\leqslant R\leqslant a, \\ 2^{3/8}\left(\frac{R-a}{1-a}\right)^{3/4}, & \text{for } a\leqslant R\leqslant a+0.3184(1-a), \\ 1-\sqrt{2}\frac{R-a}{1-a}, & \text{for } a+0.3184(1-a)\leqslant R\leqslant 0.5(a+1), \\ 1-\sqrt{1-2\left(\frac{R-1}{1-a}\right)^2}, & \text{for } 0.5(a+1)\leqslant R\leqslant 1, \end{cases}$$

$$\mu(R|\pi_5) = \begin{cases} 1-\left[1-2\left(\frac{1-R-1}{1-a}\right)^2\right]^{3/8}, & \text{for } 0\leqslant R\leqslant 0.5(1-a), \\ 1-2^{3/8}\left(\frac{1-R-a}{1-a}\right)^{3/4}, & \text{for } 0.5(1-a)\leqslant R\leqslant 1-a, \\ 1, & \text{for } 1-a\leqslant R\leqslant a, \\ 1-2^{3/8}\left(\frac{R-a}{1-a}\right)^{3/4}, & \text{for } a\leqslant R\leqslant 0.5(a+1), \\ 1-\left[1-2\left(\frac{R-1}{1-a}\right)^2\right]^{3/8}, & \text{for } 0.5(a+1)\leqslant R\leqslant 1, \end{cases}$$

$$\mu(R|\pi_6) = \begin{cases} 1-\sqrt{1-2\left(\frac{1-r-1}{1-a}\right)^2}, & \text{for } 0\leqslant R\leqslant 0.5(1-a), \\ 1-\sqrt{2}\frac{1-R-a}{1-a}, & \text{for } 0.5(1-a)\leqslant R\leqslant 0.6816(1-a), \\ 2^{3/8}\left(\frac{1-R-a}{1-a}\right)^{3/4}, & \text{for } 0.6816(1-a)\leqslant R\leqslant 1-a, \\ 0, & \text{for } 1-a\leqslant R\leqslant 1, \end{cases}$$

$$\mu(R|\pi_7) = \begin{cases} 2\left(\frac{1-R-1}{1-a}\right)^2, & \text{for } 0\leqslant R\leqslant 0.5(1-a), \\ 1-2\left(\frac{1-R-a}{1-a}\right)^2, & \text{for } 0.5(1-a)\leqslant R\leqslant 0.5630(1-a), \\ \sqrt{2}\frac{1-R-a}{1-a}, & \text{for } 0.5630(1-a)\leqslant R\leqslant 1-a, \\ 0, & \text{for } 1-a\leqslant R\leqslant 1, \end{cases}$$

$$\mu(R|\pi_8) = \begin{cases} 1-\left[1-2\left(\frac{1-R-1}{1-a}\right)^2\right]^2, & \text{for } 0\leqslant R\leqslant 0.4370(1-a), \\ 1-2\left(\frac{1-R-1}{1-a}\right)^2, & \text{for } 0.4370(1-a)\leqslant R\leqslant 0.5(1-a), \\ 2\left(\frac{1-R-a}{1-a}\right)^2, & \text{for } 0.5(1-a)\leqslant R\leqslant 1-a, \\ 0, & \text{for } 1-a\leqslant R\leqslant 1, \end{cases}$$

$$\mu(R|\pi_9) = \begin{cases} \left[1-2\left(\frac{1-R-1}{1-a}\right)^2\right]^2, & \text{for } 0\leqslant R\leqslant 0.5(1-a), \\ 4\left(\frac{1-R-a}{1-a}\right)^4, & \text{for } 0.5(1-a)\leqslant R\leqslant 1-a, \\ 0, & \text{for } 1-a\leqslant R\leqslant 1. \end{cases}$$

With the membership functions above,we can draw their curved lines again in fig.2.

The last question for us is to find out what the practical concrete method converting a accurate value of general reliability into a linguistic value of fuzzy reliability is. Naturally,we utilize the principle about maximum grade of membership in fuzzy mathematics,then we can derive the mathematical model or the method for converting we need

$$\underset{\sim}{R}(\omega) = \pi_j, \qquad \text{for} \quad \pi_j \in \mathcal{E}, \text{ and } j=1,\ldots,9,$$

$$\text{if} \quad \mu(R|\pi_j) = \max\left\{\mu(R|\pi_1),\ldots,\ \mu(R|\pi_9)\right\}.$$

2)Fuzzy Failure Rate

Fuzzy failure rate is defined as follows:fuzzy failure rate is the fuzzy linguistic probability of a product to fail in one unit time when performing a required function until a certain time point t,which represents the degree of the product not to fail according to the standard of an ideal case.

Like fuzzy reliability,we define a linguistic-valued space of fuzzy failure rate which is charactered by nine basic fuzzy subsets,that is,"extremely big failure","very big failure" , "big failure","comparatively big failure","critical failure" , "comparatively small failure","small failure","very small failure",and "extremely small failure".

Let λ_t represent an ideal failure rate to a product,and λ_r represent a real failure rate to it,then according to the value of $\frac{\lambda_r-\lambda_i}{\lambda_i}$,simply denoted by λ ,we can infer which linguistic value the fuzzy failure rate to the product is,or say, which one of the nine basic fuzzy subsets we should choose to describe the fuzzy failure to the product,for example,if $\lambda(>0)$ extremely trends to or is equal to 0,then we call the product "extremely small failure";if $\lambda(<1)$ extremely trends to or is equal to 1,then we call it "extremely big failure",and so on. Additionally,we assume that if $\lambda<0$,then the linguistic value is always "extremely small failure";if $\lambda>1$,then it is always "extremely big failure",and these two cases are so easy to be dealt with that they are beyond what we study in the sequel.

Now λ corresponds with R in fuzzy reliability,so we can using the corresponding methods in the studies of fuzzy reliability to definitions of the membership-functions of the nine basic fuzzy subsets and the rule of converting accurate values to linguistic values in the studies of fuzzy failure rate,but to make this paper short,the studies in the sequel are all omitted.

3)Fuzzy Mean Life

We define that fuzzy mean life is the value of mathematical expection of time of a product to perform a required function without any failures,which indicates the mean time before the first failure to the product,and should be represented by fuzzy linguistic value according to the standard of an ideal case.

We also define a linguistic-valued space of fuzzy mean life which is charactered by the following nine basic fuzzy subsets, "extremely long life","very long life","long life","comparatively long life","critical life","comparatively short life" , "short life","very short life",and "extremely short life".

Let E_i represent an ideal mean life to a product,and E_r represent a real mean life to it,then according to the value of $\frac{E_r}{E_i}$,simply denoted by E ,we can infer which linguistic value the fuzzy mean life to the product is,or say,which one of the nine basic fuzzy subsets we should choose to describe the fuzzy mean life to it,for example,if E extremely trends to or is equal to 0,then we call the product "extremely short life"; if E (<1) extremely trends to or is equal to 1,we call it "extremely long life". In addition,we assume that if $E > 1$, then the linguistic value is always "extremely long life", and this case is a special one which is also not in our studies.

Now we can use the corresponding methods in the studies of fuzzy reliability to study the properties of fuzzy mean life because of the correspondence of E with R ,and the studies in the sequel are also omitted in this paper.

REFERENCES

1. Zadeh,L.A., Fuzzy Sets, Information Control, V8, 338-353 , 1965.
2. Wang Peizhuang, Theory of Fuzzy Sets and Its Applications, Shanghai Science and Technology Press, 1983 .
3. Li Tingjie,Gao He, Fuzzy Reliability, BUSEFAL, issue n°35 , 158-167, July, 1988.
4. Li Tingjie,Gao He, Reliability Design, Beijing University of Aeronautics and Astronautics Press, 1983.
5. O'Connor,Patrick D.T., Practical Reliability Engineering, 2nd Edn., John Wiley & Sons Ltd., 1985.