FUZZY LOGIC NEURAL NETWORK WITH FEEDBACK

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Abstract: Dynamical and statical properties of multivariable fuzzy neural networks with feedback are investigated in this paper.

1. Introduction.

In [1] we have outlined a theory of fuzzy logic neural systems.

In this paper we aim to analyse and systhese a complex fuzzy neural network with feedback.

Dynamical and statical properties of the network will be discussed.

2. Statical model of fuzzy neural network with feedback.

Let us consider an neural network shown in Figure 1.

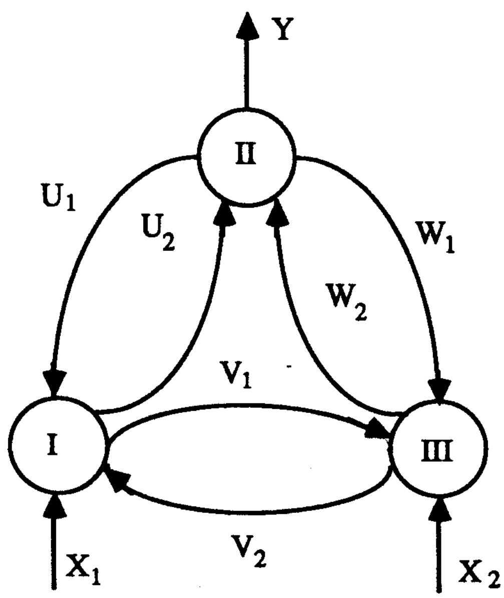

Figure 1. Neural network with feedback

The network consists of three neurons; two input signals X_1 and X_2 ; output signal Y ; and six inner feedback signals $U_1, U_2; V_1, V_2; W_1, W_2$.

According to the theory of fuzzy logic neural systems outlined in [1], [2], and [3] we may pictured a behavior of the above network by its set of fuzzy ruls and fuzzy equations:

For neuron No. I

$$\{\text{IF } X_{1(i)} \text{ and } U_{1(i)} \text{ and } V_{2(i)} \text{ then } U_{2(i)} \text{ and } V_{1(i)}\},\ i = 1, 2, 3, \dots, I$$

$$\begin{bmatrix} U_2 \\ V_1 \end{bmatrix} = \begin{bmatrix} X_1 \, U_1 \, V_2 \end{bmatrix} * \begin{bmatrix} R_1^I & R_4^I \\ R_2^I & R_5^I \\ R_3^I & R_6^I \end{bmatrix} \tag{1}$$

where $*$ = (o, Δ) - composition, and

$$R_1^I = \bigvee_i \left[X_{1(i)} \wedge U_{2(i)} \right], \ldots, R_6^I = \bigvee_i \left[V_{2(i)} \wedge V_{1(i)} \right].$$

For neuron No. II

{IF $U_{2(i)}$ and $W_{2(i)}$ then $U_{1(i)}$ and $W_{1(i)}$ and $Y_{(i)}$} , $i = 1, 2, 3, \ldots, I$

$$\begin{bmatrix} U_1 \\ W_1 \\ Y \end{bmatrix} = \begin{bmatrix} U_2 \, W_2 \end{bmatrix} * \begin{bmatrix} R_1^{II} & R_3^{II} & R_5^{II} \\ R_2^{II} & R_4^{II} & R_6^{II} \end{bmatrix} \tag{2}$$

where

$$R_1^{II} = \bigvee_i \left\{ U_{2(i)} \wedge U_{1(i)} \right\}, \ldots, R_6^{II} = \bigvee_i \left\{ W_{2(i)} \wedge Y_{(i)} \right\}$$

For neuron No. III

{IF $X_{2(i)}$ and $V_{1(i)}$ and $W_{1(i)}$ then $W_{2(i)}$ and $V_{2(i)}$}, $i = 1, 2, 3, \ldots, I$

$$W_2 = [X_2\ V_1\ W_1] * \begin{bmatrix} R_1^{III} \\ R_2^{III} \\ R_3^{III} \end{bmatrix} \qquad (3)$$

where

$$R_1^{III} = \bigvee_i [\, X_{2(i)} \wedge W_{2(i)} \,]$$

$$R_3^{III} = \bigvee_i [\, W_{2(i)} \wedge W_{2(i)} \,]$$

$$V_2 = [X_2\ V_1\ W_1] * \begin{bmatrix} R_4^{III} \\ R_5^{III} \\ R_6^{III} \end{bmatrix} \qquad (4)$$

where

$$R_4^{III} = \bigvee_i [\, X_{2(i)} \wedge W_{2\,(i)} \,]$$

$$R_6^{III} = \bigvee_i [\, W_{1(i)} \wedge V_{2(i)} \,]$$

Equations (1), (2), (3), and (4) describe a static behavior of the network in terms of fuzzy logic theory.

It is interesting to now how do the input signal X_2 and related feedback signals affect the output signal Y. Substituting Equation (3) into Equation (2) we get

$$Y = [U_2 X_2 V_1 W_1] * \begin{bmatrix} R_1^{III} \circ R_6^{II} \\ R_2^{III} \circ R_6^{II} \\ R_3^{III} \circ R_6^{II} \end{bmatrix} \quad (5)$$

Knowing X_2 and inner feedback signals U_2, V_1, and W_1 we may compute output signal Y using Equation (5).

3. Dynamical model of fuzzy neural network with feedback.

Let us consider an neural network with innter feedback shown in Figure 2.

Assume that each of the three neurons is characterized by past experience X_n and intended experience X_{n+1} [1], [2], [3].

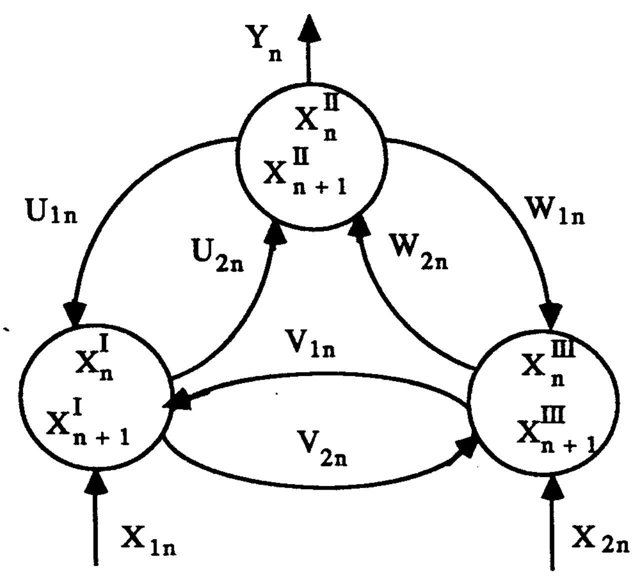

Figure 2. Neural network with feedback and experience.

A dynamic behavior of the network may be expressed by the following fuzzy rules and fuzzy equations (see Chapter 3 of [1]).

For neuron No. I

$\{\text{IF } X^I_{n(i)} \text{ and } X_{1n(i)} \text{ and } U_{1n(i)} \text{ and } V_{1n(i)} \text{ then } X^I_{n+1\,(i)} \text{ and } U_{2n(i)} \text{ and } V_{2n(i)}\}$

where

n = 0, 1, 2, 3, ... is a discrete-time variable, and i = 1, 2, 3, ... , I is the number of fuzzy rules.

$$X^I_{n+1} = \left[\, X^I_n \; X_{1n} \; U_{1n} \; V_{1n} \,\right] * \begin{bmatrix} R^{ID}_1 \\ R^{ID}_2 \\ R^{ID}_3 \\ R^{ID}_4 \end{bmatrix} \tag{6}$$

where

$$R^{ID}_1 = \bigvee_i^I \left\{ X^I_{n(i)} \wedge X^I_{n+1\,(i)} \right\}$$

$$R^{ID}_4 = \bigvee_i^I \left\{ V_{1n(i)} \wedge X^I_{n+1\,(i)} \right\}$$

$$U_{2n} = \left[\, X^I_n \; X_{1n} \; U_{1n} \; V_{1n} \,\right] * \begin{bmatrix} R^{ID}_5 \\ R^{ID}_6 \\ R^{ID}_7 \\ R^{ID}_8 \end{bmatrix} \tag{7}$$

where

$$R_5^{ID} = \bigvee_{i=1}^{I} \left\{ X_{n(i)}^{I} \wedge U_{2n(i)} \right\}$$

$$R_8^{ID} = \bigvee_{i=1}^{I} \left\{ V_{1n(i)} \wedge U_{2n(i)} \right\}$$

and

$$V_{2n} = \left[X_n^I \;\; X_{1n} \;\; U_{1n} \;\; V_{1n} \right] * \begin{bmatrix} R_9^{ID} \\ R_{10}^{ID} \\ R_{11}^{ID} \\ R_{12}^{ID} \end{bmatrix} \tag{8}$$

where

$$R_9^{ID} = \bigvee_{i=1}^{I} \left\{ X_{n(i)}^{I} \wedge V_{2n(i)} \right\}$$

$$R_{12}^{ID} = \bigvee_{i=1}^{I} \left\{ V_{1n(i)} \wedge V_{2n(i)} \right\}$$

For neuron No. II

{IF $X_{n(i)}^{II}$ and $U_{2n(i)}$ and $W_{2n(i)}$ then $X_{n+1(i)}^{II}$ and $U_{1n(i)}$ and $W_{1n(i)}$ and $Y_{n(i)}$ }

$$X_{n+1}^{II} = \left[X_n^{II} \;\; U_{2n} \;\; W_{2n} \right] * \begin{bmatrix} R_1^{II\,D} \\ R_2^{II\,D} \\ R_3^{II\,D} \end{bmatrix} \tag{9}$$

where

$$R_1^{II\,D} = \bigvee_{i=1}^{I} \left\{ X_{n(i)}^{II} \wedge X_{n+1(i)}^{II} \right\}$$

$$R_3^{II\,D} = \bigvee_{i=1}^{I} \left\{ W_{2n(i)} \wedge X_{n+1(i)}^{II} \right\}$$

$$U_{1n} = \left[\, X_n^{II} \;\; U_{2n} \;\; W_{2n} \,\right] * \begin{bmatrix} R_4^{II\,D} \\ R_5^{II\,D} \\ R_6^{II\,D} \end{bmatrix} \tag{10}$$

where

$$R_4^{II\,D} = \bigvee_{i=1}^{I} \left\{ X_{n(i)}^{II} \wedge U_{1n(i)} \right\}$$

$$R_6^{II\,D} = \bigvee_{i=1}^{I} \left\{ W_{2n(i)} \wedge U_{1n(i)} \right\}$$

and

$$W_{1n} = \left[\, X_n^{II} \;\; U_{2n} \;\; W_{2n} \,\right] * \begin{bmatrix} R_7^{II\,D} \\ R_8^{II\,D} \\ R_9^{II\,D} \end{bmatrix} \tag{11}$$

where

$$R_7^{II\,D} = \bigvee_{i=1}^{I} \left\{ X_{n(i)}^{II} \wedge W_{1n(i)} \right\}$$

$$R_9^{II\,D} = \bigvee_{i=1}^{I} \left\{ W_{2n(i)} \wedge W_{1n(i)} \right\}$$

$$Y_n = \left[\, X_n^{II} \;\; U_{2n} \;\; W_{2n} \,\right] * \begin{bmatrix} R_{10}^{II\,D} \\ R_{11}^{II\,D} \\ R_{12}^{II\,D} \end{bmatrix} \tag{12}$$

where

$$R_{10}^{II\,D} = \bigvee_{i=1}^{I} \left\{ X_{n(i)}^{II} \wedge Y_{n(i)} \right\}$$

$$R_{12}^{II\,D} = \bigvee_{i=1}^{I} \left\{ W_{2n(i)} \wedge Y_{n(i)} \right\}$$

For neuron No. III

$$\left\{ \text{IF } X^{III}_{n(i)} \text{ and } X_{2n(i)} \text{ and } V_{2n(i)} \text{ and } W_{1n(i)} \text{ then } X^{III}_{n+1(i)} \text{ and } V_{1n(i)} \text{ and } W_{2n(i)} \right\}$$

$$X^{III}_{n+1} = \left[X^{III}_{n} \; X_{2n} \; V_{2n} \; W_{1n} \right] * \begin{bmatrix} R^{III\,D}_{1} \\ R^{III\,D}_{2} \\ R^{III\,D}_{3} \\ R^{III\,D}_{4} \end{bmatrix} \tag{13}$$

where

$$R^{III\,D}_{1} = \bigvee_{i=1}^{I} \left\{ X^{III}_{n(i)} \wedge X^{III}_{n+1(i)} \right\}$$

$$R^{III\,D}_{4} = \bigvee_{i=1}^{I} \left\{ W_{1n(i)} \wedge X^{III}_{n+1(i)} \right\}$$

$$V_{1n} = \left[X^{III}_{n} \; X_{2n} \; V_{2n} \; W_{1n} \right] * \begin{bmatrix} R^{III\,D}_{5} \\ R^{III\,D}_{6} \\ R^{III\,D}_{7} \\ R^{III\,D}_{8} \end{bmatrix} \tag{14}$$

where

$$R^{III\,D}_{5} = \bigvee_{i=1}^{I} \left\{ X^{III}_{n(i)} \wedge V_{1n(i)} \right\}$$

$$R^{III\,D}_{8} = \bigvee_{i=1}^{I} \left\{ W_{1n(i)} \wedge V_{1n(i)} \right\}$$

$$W_{2n} = \left[\, X_n^{III} \;\; X_{2n} \;\; V_{2n} \;\; W_{1n} \,\right] * \begin{bmatrix} R_9^{III\,D} \\ R_{10}^{III\,D} \\ R_{11}^{III\,D} \\ R_{12}^{III\,D} \end{bmatrix} \tag{15}$$

where

$$R_9^{III\,D} = \bigvee_{i=1}^{I} \left\{ X_{n(i)}^{III} \wedge W_{2n(i)} \right\}$$

$$R_{12}^{III\,D} = \bigvee_{i=1}^{I} \left\{ W_{1n(i)} \wedge W_{2n(i)} \right\}$$

Equations (6) - (15) represent a dynamic behavior of the neural network with feedback and experience.

It is interesting to now how does the past experience X_n^{II} affect the output Y_n.

Solving Equation 9 (see Chapter 3 of [1]) we get

$$X_n^{II} = X_0^{II} \circ \left\{ R_1^{II\,D} \right\}^n \Delta \bigwedge_{i=0}^{n-1} U_{2i} \circ R_2^{II\,D} \circ \left\{ R_1^{II\,D} \right\}^{n-i-1} \Delta \bigwedge_{i=0}^{n-i} W_{1i} \circ R_3^{III\,D} \circ \left\{ R_1^{II\,D} \right\}^{n-i-1} \tag{16}$$

Substituting Equation (16) into (12) we get

$$\begin{aligned} Y_n = {} & X_0^{II} \circ \left\{ R_1^{II\,D} \right\}^n \circ R_{10}^{II\,D} \Delta \bigwedge_{i=0}^{n-1} U_{2i} \circ R_2^{II\,D} \circ \\ & \circ \left\{ R_1^{II\,D} \right\}^{n-i-1} \circ R_{10}^{II\,D} \Delta \bigwedge_{i=0}^{n-1} W_{2i} \circ \\ & \circ R_3^{II\,D} \circ \left\{ R_1^{II\,D} \right\}^{n-i-1} \circ R_{10}^{II\,D} \Delta \\ & \Delta\, U_{2n} \circ R_{11}^{II\,D} \Delta W_{2n} \circ R_{12}^{II\,D} \end{aligned} \tag{17}$$

The output Y_n constitutes of the following terms:

1) the initial experience term

$$X_0^{II} \circ \left\{ R_1^{II\,D} \right\}^n \circ R_{10}^{II\,D}$$

2) the input U_{2n} term

$$\bigwedge_{i=0}^{n-i} U_{2i} \circ R_2^{II\,D} \circ \left\{ R_1^{II\,D} \right\}^{n-i-1} \circ R_{10}^{II\,D} \circ U_{2n} \circ R_{11}^{II\,D}$$

3) the input W_{2n} term

$$\bigwedge_{i=0}^{n-1} W_{2i} \circ R_3^{II\,D} \circ \left\{ R_1^{II\,D} \right\}^{n-i-1} \circ R_{10}^{II\,D} \circ W_{2n} \circ R_{12}^{II\,D} .$$

4. Summary.

Using unified theory of fuzzy logic neural systems proposed by the authors, the dynamical and statical properties of multivariable fuzzy neural network with feedback have been discussed.

References.

[1] J.B. Kiszka, M.M. Gupta, Fuzzy logic model of single neuron, BUSEFAL, 1989.

[2] J.B. Kiszka, M.M. Gupta, Fuzzy logic neural network, BUSEFAL, 1989.

[3] J.B. Kiszka, M.M. Gupta, Fuzzy Logic neural processor, BUSEFAL, 1989.