CONTROL OF QUASILINEAR FUZZY SYSTEMS

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ABSTRACT. An approach to the problem of control of nonlinear systems via fuzzy theory is presented. The concept of QuasiLinear Fuzzy System (QLFS) is introduced and its minimal realization, stability and synthesis are discussed.

KEYWORDS. fuzzy systems, fuzzy control, fuzzy models, nonlinear systems

INTRODUCTION

Recently it was shown by Filev (1988 a,b) that the quasilinear fuzzy systems based on the fuzzy models developed by Takagi and Sugeno (1985) may be used for modelling of complex nonlinear systems as an extension of well known piecewise linear models introduced by Rajbman (1981).

In this paper the controllability ,observability and stability properties of the quasilinear fuzzy systems are discussed. An example of synthesis of dead-beat control of a nonlinear system, described by a quasilinear fuzzy model is presented.

THE QUASILINEAR FUZZY SYSTEM (QLFS).

We consider a single input - single output generally nonlinear system, which input u and/or output y and/or some of states $x_1, \ldots, x_n$ are partitioned into fuzzy sets $\mathcal{U}^j, \mathcal{Y}^k$, $\mathcal{X}_1^l, \ldots, \mathcal{X}_n^l$ in such a way that the nonlinear system S may be approximated by linear systems S_i on the fuzzy regions $\mathcal{R}_i$ defined by the cartesian products $\mathcal{R}_i = \mathcal{U}^j \times \mathcal{Y}^k \times \mathcal{X}_1^l \times \ldots \times \mathcal{X}_n^l$, $i = 1, \ldots, m$. We denote input, output and state variables, which are used to determine the fuzzy regions $\mathcal{R}_i$ as markers of the system S.

Then the model of nonlinear system S may be treated as a collection of logical relations describing fuzzy regions $\mathcal{R}_i$

and a relevant set of linear models presenting the dynamics of linear subsystems S_i, defined on $\mathcal{R}_i$:

IF u is $\mathcal{U}^i$ AND y is $\mathcal{Y}^i$ AND x_1 is $\mathcal{X}_1^i$ AND ...AND x_n is $\mathcal{X}_n^i$ THEN

$$y_i(k)=b_{0i}u(k)+\ldots+b_{1i}u(k-n)-a_{1i}y(k-1)-\ldots-a_{ni}y(k-n) \quad (1)$$

The output of S is defined is a weighed sum of subsystems S_i outputs:

$$y(k) = \sum_i v_i y_i(k) \quad (2)$$

where

$$v_i = w_i / \sum_i w_i \text{ and } w_i = \mathcal{U}^i(u) \wedge \mathcal{Y}^i(y) \wedge \mathcal{X}_1^i(x_1) \wedge \ldots \wedge \mathcal{X}_n^i(x_n), \quad (3)$$

and $\wedge$ is the **min** operation between the values of membership functions of marker fuzzy sets $\mathcal{U}^i$, $\mathcal{Y}^i$, $\mathcal{X}_j^i$. We call the system which is defined by (1)-(3) QuasiLinear Fuzzy System (QLFS).The weights v_i characterize the truth of premises of logical relations in (1). They may be interpreted geometrically as measures of marker belonging to the fuzzy regions $\mathcal{R}_i$. An equivalent state space representation of linear subsystem models in (1) is:

$$x(k+1) = A_i x(k) + B_i u(k)$$
$$y_i(k) = C_i x(k) + D_i u(k)$$

According to (2) the susbsystem S_i input-output models in (1) may be presented in a more compact form by transfer function with variable coefficients:

$$\frac{Y(z^{-1})}{U(z^{-1})} = \frac{(\Sigma v_i b_{0i}) + \ldots + (\Sigma v_i b_{ni})z^{-n}}{1 + (\Sigma v_i a_{1i}) + \ldots + (\Sigma v_i a_{ni})z^{-n}} \quad (4)$$

Equivalently for the state representation of (4) using the controllability and observability canonical forms we receive:

$$x(k+1) = A^s x(k) + B^s u(k)$$
$$y(k) = C^s x(k) + D^s u(k) \quad (5)$$

$s = c, o$

$$A^c = \begin{vmatrix} 0 & 1 & \ldots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & & 1 \\ -\Sigma v_i a_{ni} & -\Sigma v_i a_{n-1i} & \ldots & -\Sigma v_i a_{1i} \end{vmatrix} \qquad B^c = \begin{vmatrix} 0 \\ \cdot \\ \cdot \\ 1 \end{vmatrix}$$

$$C^c = \begin{vmatrix} \Sigma v_i (b_{ni} - a_{ni} b_{0i}) & \ldots & \Sigma v_i (b_{1i} - a_{1i} b_{0i}) \end{vmatrix} \qquad D^c = \Sigma v_i b_{0i}$$

$$A^{\circ}= \begin{vmatrix} -\Sigma v_i a_{1i} & 1 & 0 & \dots & 0 \\ -\Sigma v_i a_{2i} & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ -\Sigma v_i a_{ni} & 0 & 0 & \dots & 0 \end{vmatrix} \qquad B^{\circ}= \begin{vmatrix} \Sigma v_i (b_{1i} - a_{1i} b_{0i}) \\ \Sigma v_i (b_{2i} - a_{2i} b_{0i}) \\ \dots \\ \Sigma v_i (b_{ni} - a_{ni} b_{0i}) \end{vmatrix}$$

$$C^{\circ}= \begin{vmatrix} 1 & 0 & 0 & \dots & 0 \end{vmatrix} \qquad D^{\circ}= \Sigma v_i b_{0i}$$

The weights v_i in (4), (5) are defined by (3) as functions of membership of current marker values to relevant fuzzy regions $\mathfrak{R}_i$.

CONTROLLABILITY AND OBSERVABILITY PROPERTIES

From the controllability and observability canonical forms of state models is clear, that if these representations exist, i.e. subsystems S_i models are of the same order n and in minimal realization, then the controllability and observability matrices are respectively right down triangular and left down triangular full rank ones and the QLFS is in minimal realization for all values of v_i. If the minimal realizations of subsystems S_i are not of the same order, this statement will not be generally true.

SYNTHESIS

To investigate the stability of QLFS we introduce the following definition.

Definition. QLFS is stable if the input- output (4) or state space model (3) is stable for every marker value.

For some values of markers, dependent on the input or state, instability may occur as it is seen from the following example.

Example 1. We consider a nonlinear system described by by the following QLFS :

$$S(z^{-1}) = \frac{1}{1 + (7v_1 + 0.1v_2)z^{-1}}$$

For $v_1 = 0.1$; $v_2 = 0.9$ the QLFS will be stable, but for $v_1 = 0.2$; $v_2 = 0.8$ the QLFS will be unstable.

This way one has two alternatives to stabilize QLFS. The first is to ensure robust stability of subsystems S_i independently to v_i for interval of changes of the parameters. The second is in accordance with the Definition, i.e. direct stabilization of QLFS presented by (4), (5).

The synthesis of effective control of nonlinear systems described by QLFS cannot be provided independently for different virtual subsystems because of variable model parameters due to different measures of marker belonging to fuzzy regions $\mathfrak{R}_i$. This problem may be solved applying adaptive strategy only. In this paper we deal with the synthesis of time optimal control to a nonlinear system S presenting QLFS by (4) with v_i by according to (3).

Given a desired input sequence:

$$w(z^{-1}) = \frac{q(z^{-1})}{p(z^{-1})}$$

where $q(z^{-1})$, $p(z^{-1})$ are polynomials, a time optimal feedback will be designed.

From discrete linear theory is known that the solution of this problem for linear system with minimal realization $b(z^{-1})/a(z^{-1})$ is a controller with transfer function $R(z^{-1}) = m(z^{-1})/n(z^{-1})$, where

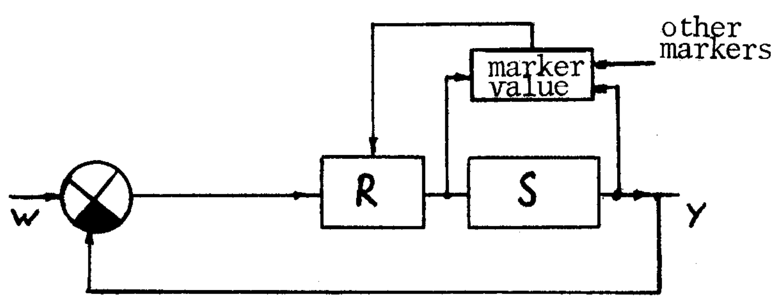
$$m = x_1/b^+q^+ \quad ; \quad n = p_0 y_1/q^+a_0^+ \qquad (6)$$

Polynomials x_1, y_1 satisfy diofantine equation:

$$b^-x + pa_0^-y = q^+$$

and $b = b^+b^-$; $a_0 = a_0^+a_0^-$; $q = q^+q^-$, a_0, p_0 coincide with polynomials a, p after cancellation of their common factors.

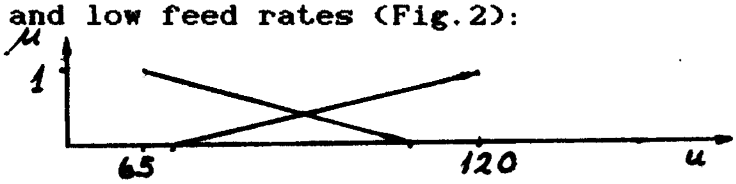
Because of variable parameters of QLFS in our case polynomials b and a in (6) are changed step by step, due to different values of markers then a recalculation of controller $r = m/n$ is needed. The structure of control system is as follows:



Example 2. We consider a typical nonlinear system - fed-batch fermentation process of ethanol production. The input-output data are presented in Tab.1.

Time	u	y	Time	u	y
1	80	0.98	15	70	5.20
2	80	1.02	16	80	6.18
3	80	1.06	17	60	6.42
4	60	1.19	18	80	5.94
5	90	1.56	19	80	7.21
6	90	1.88	20	80	7.79
7	50	2.48	21	80	7.29
8	70	2.98	22	100	7.09
9	80	3.09	23	90	8.01
10	80	3.84	24	130	8.18
11	70	4.49	25	120	8.15
12	70	4.48	26	100	8.17
13	80	5.25	27	120	9.04
14	70	5.48	28	120	8.68

Only one marker is used in this example - u(k-1), characterizing the two working regions of system - at high feed rates and low feed rates (Fig.2):



The old value of input u(k-1) is used as a marker in order to respect slow dynamics of system. The QLFS obtained after identification is:

$$S(z^{-1}) = \frac{-0.54v_1+2.75v_2+(0.89v_1-1.28v_2)z^{-1}}{1+(-0.01v_1+0.05v_2)z^{-1}}$$

For every v_1 and v_2 the denominator polynomial is stable. It is not true for the nominator. Let $w(z^{-1}) = 10 + 3z^{-1}$ is the desired sequence to be followed. Applying (6) for controller transfer function we receive:

$$R(z^{-1}) = \frac{1+(-0.01v_1+0.05v_2)z^{-1}}{-0.54v_1+2.75v_2+10+(0.89v_1-1.28v_2+3)z^{-1}}$$

CONCLUSION

The concept of control of nonlinear systems via QLFS may be extended to the solution of further control problems -

optimal control, multivariable control, tracking problem and so on.

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