SOME PROPERTIES FOR GREY CONTINUOUS MAPPING

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ABSTRACT: In this paper some properties of grey continuous mapping from a grey topological space to a grey topological space is studied.

KEYWORDS: Grey continuous mapping, grey point, grey open set and grey close set.

I. INTRODUCTION

We gave the definition and some properties of grey continuous mapping in [2] and [3]. We shall study other properties of grey continuous mapping on this basis.

Theorem 1: Let f be a mapping from the grey topological space $(X,\mathfrak{I})$ to the grey topological space $(Y,\mathfrak{I}')$.

(1) If A is a grey subset of X and B is a grey subset of Y,

then $\bar{\mu}_{f(A)}(y)\leqslant \bar{\mu}_B(y)$, $\underline{\mu}_{f(A)}(y)\leqslant \underline{\mu}_B(y)$, $\forall y\in Y$.

$\bar{\mu}_A(x)\leqslant \bar{\mu}_{f^{-1}(B)}(x)$, $\underline{\mu}_A(x)\leqslant \underline{\mu}_{f^{-1}(B)}(x)$, $\forall x\in X$.

(2) If $At(t\in T)$ are grey subsets of X, then

$$f[\bigcup_{t\in T} At]=\bigcup_{t\in T} f[At],\quad f[\bigcap_{t\in T} At]=\bigcap_{t\in T} f[At].$$

(3) If $Bt(t\in T)$ are grey subsets of Y, then

$$f^{-1}[\bigcup_{t\in T} Bt]=\bigcup_{t\in T} f^{-1}[Bt],\quad f^{-1}[\bigcap_{t\in T} Bt]=\bigcap_{t\in T} f^{-1}[Bt].$$

Definition 1: Let $(X,\mathfrak{I})$ be a grey topological space, $\mathcal{B}\subseteq\mathfrak{I}$ is a subfamily of $\mathfrak{I}$. If every grey open set is a union of some members of $\mathcal{B}$, then $\mathcal{B}$ is called base of $\mathfrak{I}$.

If $\mathcal{B}'$ is a subfamily of $\mathfrak{I}$ and every member of $\mathcal{B}$ is a intersection of finite grey set of $\mathcal{B}'$, then $\mathcal{B}'$ is called subbase of $\mathfrak{I}$.

II. THE PROPERTIES OF GREY CONTINUOUS MAPPING

Theorem 2: Let f be a mapping from the grey topological space $(X,\mathfrak{I})$ to the grey topological space $(Y,\mathfrak{I}')$, then following are equivalent:

(1) f is the grey continuous mapping.

(2) The inverse image $f^{-1}[B]$ of grey close set B in Y is the grey close set in X.

(3) There exists subbase $\mathcal{B}$ of topology $\mathfrak{T}'$ in Y such that inverse image $f^{-1}[C]$ of every member C of it is the grey open set of topology $\mathfrak{T}$ in X.

(4) The inverse image $f^{-1}[B]$ of neighbourhood B of image f(a) of every grey point a in X is neighbourhood of grey point a.

(5) If choose neighbourhood B of image f(a) of every grey point a in X, then always there exists a neighbourhood A of a such that $\bar{\mu}_{f[A]}(y) \leq \bar{\mu}_B(y)$, $\underline{\mu}_{f[A]}(y) \leq \underline{\mu}_B(y)$, $\forall y \in Y$.

Proof: From [3] we have (1)→(2).

If (2) is correct, B is a grey close set in Y and $f^{-1}[B]$ is a grey close set in X, then B^c is the grey open set in Y(or $B^c \in \mathfrak{T}'$) and $(f^{-1}[B])^c$ is the grey open set in X. Hence $f^{-1}[B^c] = (f^{-1}[B])^c$ is the grey open set in X(or $f^{-1}[B^c] \in \mathfrak{T}$).

From [2] we have f is the grey continuous mapping.

So (2)→(1).

Hence (1)←→(2).

Since f is the grey continuous mapping, hence $\forall B \in \mathfrak{T}' \rightarrow f^{-1}[B] \in \mathfrak{T}$. Also $\mathcal{B} \subseteq \mathfrak{T}'$, thus $\forall C \in \mathcal{B} \subseteq \mathfrak{T}' \rightarrow f^{-1}[C] \in \mathfrak{T}$. Hence there exists subbase $\mathcal{B}$ of topology $\mathfrak{T}'$ in Y such that inverse image $f^{-1}[C]$ of every member C of it is the grey open set of topology $\mathfrak{T}$ in X.

So (1)→(3).

Since $\mathcal{B}$ is the subbase of $\mathfrak{T}'$, thus every grey open set of is a union of intersection of some finite grey set of $\mathcal{B}$ (or $\forall D \in \mathfrak{T}$, $\exists Di(i=1,2,\ldots,n) \in \mathcal{B}$, s.t. $D = \cup(\cap Di)$).

From theorem1(3) we have $f^{-1}[\cup(\cap Di)] = \cup\cap(f^{-1}[Di])$.

If (3) is correct, then inverse image $f^{-1}[Di]$ of members Di of $\mathcal{B}$ are grey open sets of $\mathfrak{T}$ in X (or $f^{-1}[Di] \in \mathfrak{T}$).

Hence $f^{-1}[D] = f^{-1}[\cup(\cap Di)] = \cup\cap(f^{-1}[Di]) \in \mathfrak{T}$, thus f is the grey continuous mapping.

So (3)$\rightarrow$(1).

Hence (1)$\leftarrow\rightarrow$(3).

Choose any grey point $a \in X$ and any neighbourhood $B \in \mathcal{U}_{f(a)}$. From the definition of neighbourhood we have there exists $C \in \mathfrak{J}'$ such that $f(a) \in C \subseteq B$. Hence $a \in f^{-1}[C] \subseteq f^{-1}[B]$.

Also f is the grey continuous mapping, then $f^{-1}[C] \in \mathfrak{J}$. Hence there exists $f^{-1}[C] \in \mathfrak{J}$ such that $a \in f^{-1}[C] \subseteq f^{-1}[B]$, thus $f^{-1}[B] \in \mathcal{U}_a$. Hence the inverse image $f^{-1}[B]$ of neighbourhood B of image f(a) of every grey point a in X is neighbourhood of grey point a.

So (1)$\rightarrow$(4).

Choose any $a \in X$ and $B \in \mathcal{U}_{f(a)}$. From (4) we have $f^{-1}[B] \in \mathcal{U}_a$. From [3] we have $\bar{\mu}_{f[f^{-1}[B]]}(y) \leq \bar{\mu}_B(y), \underline{\mu}_{f[f^{-1}[B]]}(y) \leq \underline{\mu}_B(y)$, $\forall y \in Y$. Let $f^{-1}[B]=A$, then $\forall a \in X$ and $\forall B \in \mathcal{U}_{f(a)}$, there exists $A \in \mathcal{U}_a$ such that $\bar{\mu}_{f[A]}(y) \leq \bar{\mu}_B(y)$, $\underline{\mu}_{f[A]}(y) \leq \underline{\mu}_B(y)$, $\forall y \in Y$.

So (4)$\rightarrow$(5).

Let B be a grey open set in Y (or $B \in \mathfrak{J}'$) and $I=\{x \in X \mid \underline{\mu}_{f^{-1}[B]}(x)>0\}$.

Choose any $x \in X$ and let $\bar{\mu}_{f^{-1}[B]}(x)=\bar{\lambda}$, $\underline{\mu}_{f^{-1}[B]}(x)=\underline{\lambda}$, and $y=f(x)$. Then $f(x_{\bar{\lambda}\underline{\lambda}})=y_{\bar{\lambda}\underline{\lambda}}$, hence $y_{\bar{\lambda}\underline{\lambda}} \in B$ and B is the open neighbourhood of $y_{\bar{\lambda}\underline{\lambda}}$.

From (5) we have there exists open neighbourhood $A \in \mathcal{U}_{x_{\bar{\lambda}\underline{\lambda}}}$ such that $\bar{\mu}_{f[A]}(y) \leq \bar{\mu}_B(y)$, $\underline{\mu}_{f[A]}(y) \leq \underline{\mu}_B(y)$, $\forall y \in Y$.

Hence all there exists open neighbourhood $A \in \mathcal{U}_{x_{\bar{\lambda}\underline{\lambda}}}$ $\forall x \in I$.

Let $C=\bigcup A$, then $\bar{\mu}_C(x) \geq \bar{\mu}_{f^{-1}[B]}(x), \underline{\mu}_C(x) \geq \underline{\mu}_{f^{-1}[B]}(x)$, $\forall x \in X$.

Also choose any $x \in I$, there exists open neighbourhood A of x such that $\bar{\mu}_{f[A]}(y) \leq \bar{\mu}_B(y), \underline{\mu}_{f[A]}(y) \leq \underline{\mu}_B(y)$, $\forall y \in Y$. From theorem1(2) we have $f[C]=f[\bigcup A]=\bigcup f[A]$. Hence

$$\bar{\mu}_{f[C]}(y) \leq \bar{\mu}_B(y),\ \underline{\mu}_{f[C]}(y) \leq \underline{\mu}_B(y),\ \forall y \in Y.$$

From theorem1(1) we have $\bar{\mu}_C(x) \leq \bar{\mu}_{f^{-1}[B]}(x), \underline{\mu}_C(x) \leq \underline{\mu}_{f^{-1}[B]}(x)$, $\forall x \in X$. Hence $\bar{\mu}_C(x)=\bar{\mu}_{f^{-1}[B]}(x)$, $\underline{\mu}_C(x)=\underline{\mu}_{f^{-1}[B]}(x)$, $\forall x \in X$.

So $C=\bigcup A=f^{-1}[B]$.

Also A is the grey open set, thus $f^{-1}[B]$ is the grey open set. From [2] we have f is the grey continuous mapping.

So (5)→(1).

Theorem 3: Let f be a mapping from the grey topological space $(X, \mathfrak{J})$ to the grey topological space $(Y, \mathfrak{J}')$. If f is the grey continuous mapping, then given any coinciderce field $B \in \mathcal{U}_{f(a)}$ of image f(a) of every grey point a in X, always there exists coinciderce field $A \in \mathcal{U}_a$ of a in X such that

$$\bar{\mu}_{f[A]}(y) \leq \bar{\mu}_B(y), \ \underline{\mu}_{f[A]}(y) \leq \underline{\mu}_B(y), \forall y \in Y.$$

Proof: Choose any grey point $a = x_{\bar{\lambda}\underline{\lambda}} \in X$, and $B \in \mathcal{U}_{f(a)}$ is any coinciderce field of f(a). From the definition of coinciderce field we have there exists $C \in \mathfrak{J}'$ such that $f(a) \triangle C \subseteq B$.

Let $A = f^{-1}[C]$. Since f is the grey continuous mapping, hence $A \in \mathfrak{J}$.

Let $y = f(x)$, then $\underline{\mu}_A(x) = \underline{\mu}_{f^{-1}[C]}(x) = \underline{\mu}_C(f(x)) = \underline{\mu}_C(y) > 1 - \underline{\lambda}$.
$\rightarrow \underline{\mu}_A(x) + \underline{\lambda} > 1$. Hence $a \triangle A$, so $A \in \mathcal{U}_a$ is the coinciderce field of a. And from [3] we have

$$\bar{\mu}_{f[A]}(y) = \bar{\mu}_{f[f^{-1}[C]]}(y) \leq \bar{\mu}_B(y), \underline{\mu}_{f[A]}(y) = \underline{\mu}_{f[f^{-1}[C]]}(y) \leq \underline{\mu}_B(y), \forall y \in Y.$$

Hence given any coinciderce field $B \in \mathcal{U}_{f(a)}$ of image f(a) of every grey point a in X, always there exists coinciderce field $A \in \mathcal{U}_a$ of a in X such that

$$\bar{\mu}_{f[A]}(y) \leq \bar{\mu}_B(y), \underline{\mu}_{f[A]}(y) \leq \underline{\mu}_B(y), \quad \forall y \in Y.$$

Theorem 4: Let f be a mapping from the grey topological space $(X, \mathfrak{J})$ to the grey topological space $(Y, \mathfrak{J}')$, B is a grey subset of Y. If for any $B \in \mathfrak{J}'$ there always hold

$$\bar{\mu}_{\overline{f^{-1}[B]}}(x) \leq \bar{\mu}_{f^{-1}[\bar{B}]}(x), \ \underline{\mu}_{\overline{f^{-1}[B]}}(x) \leq \underline{\mu}_{f^{-1}[\bar{B}]}(x), \forall x \in X.$$

Then f is the grey continuous mapping.

Proof: Suppose B is the grey close set of Y, then $\bar{B} = B$. Also $\bar{\mu}_{\overline{f^{-1}[B]}}(x) \leq \bar{\mu}_{f^{-1}[\bar{B}]}(x)$, $\underline{\mu}_{\overline{f^{-1}[B]}}(x) \leq \underline{\mu}_{f^{-1}[\bar{B}]}(x)$, $\forall x \in X$. Hence
$\bar{\mu}_{\overline{f^{-1}[B]}}(x) \leq \bar{\mu}_{f^{-1}[\bar{B}]}(x) = \bar{\mu}_{f^{-1}[B]}(x)$, $\underline{\mu}_{\overline{f^{-1}[B]}}(x) \leq \underline{\mu}_{f^{-1}[\bar{B}]}(x) = \underline{\mu}_{f^{-1}[B]}(x)$, $\forall x \in X$.

Also it is obvious $\bar{\mu}_{\overline{f^{-1}[B]}}(x) \geq \bar{\mu}_{f^{-1}[B]}(x)$, $\underline{\mu}_{\overline{f^{-1}[B]}}(x) \geq \underline{\mu}_{f^{-1}[B]}(x)$

$\forall x \in X$. Hence $\bar{\mu}_{\overline{f^{-1}[B]}}(x)=\bar{\mu}_{f^{-1}[B]}(x)$, $\underline{\mu}_{\overline{f^{-1}[B]}}(x)=\underline{\mu}_{f^{-1}[B]}(x)$, $\forall x \in X$. Then $\overline{f^{-1}[B]}=f^{-1}[B]$. So $f^{-1}[B]$ is the grey close set of X. From theorem2 we have f is the grey continuous mapping.

Corollary: Let f be a mapping from the grey topological space $(X,\mathfrak{I})$ to the grey topologict space $(Y,\mathfrak{I}')$, A is a grey subset of X. If for any $A\in\mathfrak{I}$ there always hold

$$\bar{\mu}_{f[\bar{A}]}(y)\leqslant\bar{\mu}_{\overline{f[A]}}(y),\ \underline{\mu}_{f[\bar{A}]}(y)\leqslant\underline{\mu}_{\overline{f[A]}}(x),\ \forall x\in X.$$

Then f is the grey continuous mapping.

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