A CONTRIBUTION TO HYPERSCALE - BASED HYPERDOMINANCE AND MULTIDIMENSIONAL DISCREPANCY ANALYSIS USING ORDERING FUNCTIONS OF FUZZY SUBSETS OF [0,1] .

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ABSTRACT : This paper uses hyperscales as a basis for hyperdominance analysis and employs the normalized possibilistic measure of nonspecificity proposed by Higashi and Klir in [14] to assess the inter-fuzzy subsets overall discrepancy. Also, it introduces object evaluation via what the author calls somewhat objective fuzzy appreciations (**SOFAs**) .

KEYWORDS : Entropy-based weight of importance, Fuzzy Integral, Hyperdominance, Hyperrelation, Hyperscale, Level 2 Fuzzy Set Reduction, m-Flou Set, Possibilistic Measure of Nonspecificity .

1-INTRODUCTION

Let's assume the giving of a descriptor set. When it comes to objectively appreciating various objects belonging to a given object universe using the aggregation operators developed in fuzzy set literature, one is to face up, inevitably, with the serious problem of choosing the appropriate aggregation function . In fact, whatever aggregation function one decides to use, it reflects, undoubtedly, some psychological traits of the decision maker's or analyst's personality : optimism, pessimism or any sort of involvement in the choice process of the very relationship in question between the descriptors . Consequently, a kind of fuzziness stems from the ambiguity surrounding the determination of the right aggregation function, an ambiguity felt in some circumstances and brought about by a serious desire of making rather objective appreciations of objects . That is, appreciations that need not reflect the decision maker's or analyst's personality, hence one may be puzzled over the appropriate aggregation function to use . Thus, in order to define different hyperrelations (generalized binary relations) of dominance [1] in a collection of fuzzy subsets, we suggest herein the usage of a very specific type of fuzzy appreciations that take into account a continuum of values obtained by various individual aggregation functions considered separately . Besides, we motivate the usage of the normalized possibilistic measure of nonspecificity proposed by Higashi and Klir [14] to assess the inter-subsets overall discrepancy when using a hyperscale as a basis for hyperdominance . Throughout this paper, $\mathbf{\Omega}$ will denote an object sample set taken from an object universe $\mathbf{U}$ finite or not, $\mathbf{C}$ a collection of fuzzy subsets of $\mathbf{\Omega}$, and $\mathbf{D}$ a descriptor set . In [20] we have already mentioned that once given an object by descriptor nonfuzzy data matrix, it was possible to obtain a metric information matrix called texture matrix according to table 1, below . $\underset{\sim}{X} = \sum_{d \in D} \mu_{\underset{\sim}{X}}(d)/d$ will, then denote the fuzzy profile of an object X relative to a given texture matrix.

TABLE 1

Descriptor	Anchor value	ratio scaling	interval scaling
Benefit indicator	Maximum	$\mu_{\underset{\sim}{X}}(d) = \frac{d(x)}{d^{**}}$	$\mu_{\underset{\sim}{X}}(d) = \frac{d(x)-d^{*}}{d^{**}-d^{*}}$
Cost indicator	Minimum	$\mu_{\underset{\sim}{X}}(d) = \frac{d^{*}}{d(x)}$	$\mu_{\underset{\sim}{X}}(d) = \frac{d^{**}-d(x)}{d^{**}-d^{*}}$
Coombs' ideal value	$\mu_{\underset{\sim}{X}}(d) = \left(\frac{1}{2} \left[\frac{d(x)}{d_c} + \frac{d_c}{d(x)} \right] \right)^{-1}$		

N.B. : d(x) expresses the score of an object X with respect to a descriptor d, d^{**} (resp. d^{*}) stands for maximum (resp. minimum) of the scores attained by the various objects and d_c is the Coombs' ideal value for descriptor d [26, pp.159,160].

2 - HYPERSCALE-BASED HYPERDOMINANCE

With a view to defining hyperrelations of dominance on the collection $\mathfrak{C}$, we first need to define appreciation functions of the form :

$$a(X) = f(\lambda, \underset{\sim}{X}) \qquad (1)$$

where f is a function combining the components of the descriptors importance vector λ with the fuzzy profile elements of object X. It is possible to define f in various ways.

2.1- SINGLE-VALUED APPRECIATIONS

2.1.1- METRIC-BASED APPRECIATIONS

If λ is a probabilistic weighting vector (i.e., $\lambda_d \geq 0$ and $\sum_{d \in D} \lambda_d = 1$) then, Eq.(2) defines a family of metric-based appreciations ;

$$a_p(X) = 1 - \left(\sum_{d \in D} \lambda_d^p \left[1 - \mu_{\underset{\sim}{X}}(d) \right]^p \right)^{\frac{1}{p}} \quad , 1 \leq p \leq +\infty \qquad (2)$$

and it follows that for all $X \in \Omega$ and all $p \in [1, +\infty[$, the appreciations $a_p(X)$ satisfy the following properties :

i) $0 \leq a_p(X) \leq 1$; and

ii) $a_p(X) \geq a_p(Y)$ if $\underset{\sim}{X} \geq \underset{\sim}{Y}$.

It is to be noted that the probabilistic weighting vector λ may be transformed into a normalized possibilistic weighting vector Π (i.e., $\Pi_d \geq 0$

and $\max_{d \in D} \Pi_d = 1$) by means of Eq. (3) : $\Pi_d = \sum_{d' \in D} \min(\lambda_d, \lambda_{d'})$ (3)

(see [13,19], also see [2, pp.169]), so another type of metric-based appreciations a(X) given by Eq. (4) may be considered.

$$a(X) = 1 - \sup_{d \in D} [(1 - \mu_{\underline{X}}(d)) \wedge \Pi_d] \quad (4)$$

2.1.2-FUZZY SET AGGREGATION CONNECTIVE BASED APPRECIATIONS

In fuzzy set literature, several fuzzy set aggregation connectives : triangular (co)norms, averaging operators, compensatory operators and self-dual operators are surveyed [12,15,16]. Details of n-ary and probabilistically or possibilistically weighted generalizations of some of these operators may be found in [2] . Examples of these generalizations are shown in table2, below .

TABLE 2

Aggregation function	n-ary generalization	weighted generalization
$(x_1+x_2-1) \vee 0$	$[x_1+x_2+...+x_n-(n-1)] \vee 0$	$[n(p_1x_1+..+p_nx_n)-(n-1)] \vee 0$
$x_1 . x_2$	$x_1.x_2...x_n$	$x_1^{np_1}...x_n^{np_n}$
$x_1 \wedge x_2$	$\min(x_1, x_2,..., x_n)$	$[x_1 \vee (1-\pi_1)] \wedge ... \wedge [x_n \vee (1-\pi_n)]$
$(x_1+x_2)/2$	$(x_1+x_2+...+x_n)/n$	$p_1x_1+p_2x_2+...+p_nx_n$
$x_1 \vee x_2$	$\max(x_1, x_2,..., x_n)$	$[x_1 \wedge \pi_1] \vee ... \vee [x_n \wedge \pi_n]$
$x_1+x_2-x_1.x_2$	$1-(1-x_1).(1-x_2)...(1-x_n)$	$1-(1-x_1)^{np_1}...(1-x_n)^{np_n}$
$g^{-1}(g(x_1)+g(x_2))$	$g^{-1}(g(x_1)+...+g(x_n))$	$g^{-1}(n[p_1g(x_1)+...+p_n g(x_n)])$

<u>**N.B**</u> : g is an additive generator of an archimedean aggregation operator.

The aggregation functions may be chosen on the basis of a relationship between the descriptors involved i.e., competitiveness and compensation as shown in table 3, below ([23], also see [26, pp. 324, 325]) or on the basis of some psychological traits of the decision maker's personality, namely, a characteristic optimism (pessimism) degree [17] See Eq.(5).

$$a_s(X) = [\sum_{d \in D} \lambda_d \, . \, (\mu_{\underline{X}}(d))^s]^{1/s} \quad (5)$$

where s is a characteristic optimism index . By varying s, various aggregation operators are obtained . For instance, if $s \to -\infty$, we get the min-operator, the lowest values are, thus, dominant, this corresponds to a pessimistic aggregation . If $s \to +\infty$, we get the max-operator,the highest values are, thus, dominant, this corresponds to an optimistic aggregation . In [17], transformations are suggested to make the characteristic optimism index take on its values in the valuation set [0,1] .

TABLE 3

Descriptors relationship	Appreciation formula
competitive and noncompensatory	$a(X) = \min_{d \in \mathfrak{D}} \mu_{\underset{\sim}{X}}(d)$
competitive and compensatory	$a(X) = \perp_{d \in \mathfrak{D}} \mu_{\underset{\sim}{X}}(d)$
noncompetitive and noncompensatory	$a(X) = \max_{d \in \mathfrak{D}} \mu_{\underset{\sim}{X}}(d)$
noncompetitive and compensatory	$a(X) = *_{d \in \mathfrak{D}} \mu_{\underset{\sim}{X}}(d)$

N.B : $\perp$ stands either for product operator : $a \perp b = a \,.\, b$ or bold intersection operator : $a \perp b = \max(0, a+b-1)$ and $*$ stands either for the bold union operator : $a*b = \min(1, a+b)$ or the probabilistic sum operator : $a*b = a+b-ab$.

2.1.3- OWA-OPERATOR BASED APPRECIATIONS

If $\mathfrak{D} = \{ d_1, d_2, ..., d_n \}$ is the descriptor set and R is an OWA operator with weighting vector W [24], the appreciation a(X) will be stated as :

$$a(X) = R(e_1, \ldots, e_n) \qquad (6)$$

where $e_i = H(\mu_{\underset{\sim}{X}}(d_i), \lambda_{d_i})$. If b_k denotes the k^{th} largest element in the bag $\langle e_1, \ldots, e_n \rangle$, then a(X) will be : $a(X) = \sum_{k=1}^{k=n} b_k.W_k$ (7)

In [24] Yager suggested inter alia the following form of the function H :

$$e_i = H(\mu_{\underset{\sim}{X}}(d_i), \lambda_{d_i}) = (\lambda_{d_i} \vee p) \,.\, [\mu_{\underset{\sim}{X}}(d_i)]^{(\lambda_{d_i} \vee q)} \qquad (8)$$

where q is the degree of orness associated with W and p its complement, i.e., $p+q = 1$ and $q = \left[\frac{1}{n-1}\right] \sum_{i=1}^{i=n} (n-i).W_i$ (9)

2.1.4- FUZZY INTEGRAL BASED APPRECIATIONS

Following Wierzchon [22] in his interpretation of Sugeno's fuzzy integral [21], we will be able to evaluate any given object X, as follows : we will let $\mu_{\underset{\sim}{X}}(d)$ express the grade of satisfaction provided by object X, if descriptor d is considered, then, if E is a subset of $\mathfrak{D}$, the best security grade of satisfaction provided by object X will be : $s(E) = \min_{d \in E} \mu_{\underset{\sim}{X}}(d)$ and in view of the transformation mentioned before, the possibilistic importance measure of the descriptor subset E will be given by $w(E) = \max_{d \in E} \Pi_d$. The value v(X) given by : $v(X) = \max_{E \subset \mathfrak{D}} [s(E) \wedge w(E)]$ (10) is known as the best pessimistic evaluation, whereas, the worst optimistic evaluation will be : $r(X) = \min_{E \subset \mathcal{D}} [\, s(E) \vee w(E) \,]$ (11)

In [10, pp. 138,139], the formula (10) is stated as :

$$v(X) = \max_{d \in \mathcal{D}} \min [\, \mu_{\underline{X}}(d), \Pi_d \,] \qquad (12)$$

and according to Wierzchon [22] the formula (11) can be rewritten into :

$$r(X) = v(x) + |\, \mu_{\underline{X}}(d_{i_0}) - \max_{1 \le i \le i_0} \Pi_{d_i} \,| \qquad (13)$$

provided that the $\mu_{\underline{X}}(d_i)$s are decreasingly ordered and d_{i_0} is such that :

$$v(X) = \mu_{\underline{X}}(d_{i_0}) \wedge \max_{1 \le i \le i_0} \Pi_{d_i} \qquad (14)$$

2.1.5- HYPERSCALE FORMULATION

At this point, we are capable of assigning to each fuzzy subset $\mathbf{B} = \sum \mu_{\mathbf{B}}(X)/X$, belonging to the collection $\mathbf{C}$, a discrete fuzzy subset

$\underline{A}(\mathbf{B}) = \sum \mu_{\mathbf{B}}(X)/a(X)$ of the unit interval [0,1]. Consequently, the ranking of

the different fuzzy subsets belonging to $\mathbf{C}$ boils down, to a ranking of the corresponding discrete fuzzy subsets $\underline{A}(\mathbf{B})$ of [0,1]. A hyperscale Ψ (scalar function) may be used so as to rank these fuzzy subsets. Indeed, if F is an ordering function of fuzzy subsets of [0,1] [3, 25], we will set :

$$\Psi(\mathbf{B}) = F[\, \underline{A}(\mathbf{B}) \,] \qquad (15)$$

2.2- SOMEWHAT OBJECTIVE FUZZY APPRECIATIONS (SOFAs)

In real-world situations, the human aggregation schemes result in appreciations which lie in between pessimistic appreciations and optimistic ones. Moreover, these appreciations are neither totally pessimistic nor totally optimistic [27] . Inspired from these empirical findings we will employ the following basic result in order to make the definition and usage of SOFAs possible .

***BASIC RESULT*: Let $P_*(X)$ be the pessimistic appreciation of an object X, $P^*(X)$ its best pessimistic appreciation, $O_*(X)$ its worst optimistic appreciation and $O^*(X)$ its optimistic appreciation, then the following inequalities hold :**

$$P_*(X) \le P^*(X) \le O_*(X) \le O^*(X) \qquad (16)$$

The values $P_*(X)$ and $O^*(X)$ will be referred to as the extreme appreciations, and $P^*(X)$ and $O_*(X)$ as the intermediate ones . It is to be stressed that the normalized possibilistic weighting vector Π used in the calculus of $P^*(X)$ and $O_*(X)$ involved in the formulation of the various

SOFAs, results from the transformation of the informational probabilistic weighting vector λ, representing the descriptors entropy-based weights of importance calculated on the basis of a given texture matrix. That is,

$$\lambda_d = \frac{1-e(d)}{n-E} \qquad (17)$$

where e(d) is the entropy measure of descriptor d contrast intensity and E is the total entropy (i.e., $E = \sum_{d \in D} e(d)$). See [26, pp.188,189]. Hereafter, we introduce the main types of SOFAs, precisely, interval-valued SOFAs, fuzzy interval-valued SOFAs and m-flou set-valued SOFAs (m = 2).

2.2.1- INTERVAL- VALUED SOFAs

Let $\mathbf{B} = \sum_{i=1}^{i=k} g_i/X_i$ be a fuzzy subset belonging to $\mathbf{C}$, to each object X belonging to supp$\mathbf{B}$ it is possible to assign, an interval-valued SOFA :

$$\underline{a}(X) = [\ \mathbf{P}_*(X), \mathbf{O}^*(X)\ ] \qquad (18)$$

if the two extreme appreciations are considered or alternatively :

$$\overline{a}(X) = [\ \mathbf{P}^*(X), \mathbf{O}_*(X)\ ] \qquad (19)$$

if the intermediate ones are considered.

Once given the different interval-valued SOFAs of the various objects belonging to supp$\mathbf{B}$, we start by transforming the possibilistic vector of membership grades into a probabilistic vector by means of Eq.(20) i.e., we set p_i equal to : $p_i = \sum_{J=i}^{J=k} \frac{\pi_J - \pi_{J+1}}{J}$ (20)

where $\pi_1,\ldots,\pi_k$ and $\pi_{k+1} = 0$, are the normalized and decreasingly ordered membership grades g_i for i=1,...,k ([13,19], also see [2, pp. 168]). If F is an ordering function of fuzzy subsets of [0,1], the hyperscale Ψ will be defined by :

$$\Psi(\mathbf{B}) = F(\ [\ \sum_{1\le i\le k} p_i \,.\, \mathbf{P}_*(X_i),\ \sum_{1\le i\le k} p_i \,.\, \mathbf{O}^*(X_i)\ ]\ ) \qquad (21)$$

if the two extreme appreciations are involved or alternatively by :

$$\Psi(\mathbf{B}) = F(\ [\ \sum_{1\le i\le k} p_i \,.\, \mathbf{P}^*(X_i),\ \sum_{1\le i\le k} p_i \,.\, \mathbf{O}_*(X_i)\ ]\ ) \qquad (22)$$

if the two intermediate ones are involved. The result for each fuzzy subset $\mathbf{B}$ is an interval of the form : $\underline{A}(\mathbf{B}) = [\ L(\mathbf{B}), U(\mathbf{B})\ ]$, and it is possible to rank the fuzzy subsets according to their evaluations using Ponsard's linear ordering defined on the set of intervals of [0,1] ([18], also see [10, pp. 65]) or using the hyperscale Ψ.

2.2.2- FUZZY -INTERVAL-VALUED SOFAs

For each object X, the values $\mathbf{P}_*(X)$, $\mathbf{P}^*(X)$, $\mathbf{O}_*(X)$ and $\mathbf{O}^*(X)$ may be used as characteristic points of a fuzzy interval when using Buckley's notation [4], that is, [$\mathbf{P}_*(X)$, $\mathbf{O}^*(X)$] will be the fuzzy interval's carrier and [$\mathbf{P}^*(X)$, $\mathbf{O}_*(X)$] its core . See figure. F .

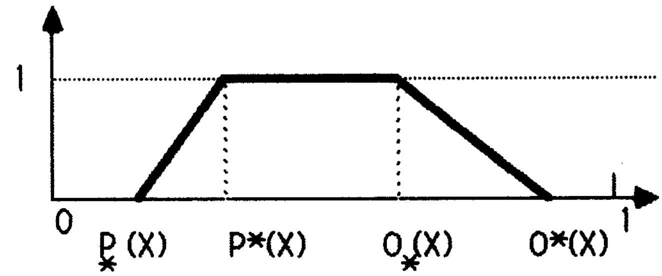

Figure. F

Thus we can assign to each object X a fuzzy interval-valued SOFA defined by : $\tilde{a}(X) = (\mathbf{P}_*(X) \,/\, \mathbf{P}^*(X), \mathbf{O}_*(X) \,/\, \mathbf{O}^*(X))$ (23)

then we can obtain a fuzzy evaluation of each fuzzy subset $\mathfrak{B}$ by using a linear aggregation scheme [9] or a level 2 fuzzy set reduction ([5,9] see also [10, pp. 62,63] .

2.2.2.1- LINEAR AGGREGATION SCHEME

Once given the different fuzzy interval-valued SOFAs of the various objects belonging to the fuzzy subset $\mathfrak{B}$, we transform the possibilistic vector of membership grades into a probabilistic vector p . The hyperscale Ψ will, then, be defined by Eq. (24) :

$$\Psi(\mathfrak{B}) = F\Big(\Big(\sum_{1\le i\le k} p_i \mathbf{P}_*(X_i) \,/\, \sum_{1\le i\le k} p_i\, \mathbf{P}^*(X_i),\ \sum_{1\le i\le k} p_i \mathbf{O}_*(X_i) \,/\, \sum_{1\le i\le k} p_i \mathbf{O}^*(X_i)\Big)\Big) \quad (24)$$

where F is, once more an ordering function . The result for each fuzzy subset of the collection $\mathfrak{C}$ is a fuzzy interval, and the fuzzy subsets could be ranked according to their evaluations using necessity or possibility measures [11], maximizing set and minimizing set [7], fuzzy relations [8] , level comparison based fast method [6] ,..., or the hyperscale Ψ .

2.2.2.2 - LEVEL 2 FUZZY SET REDUCTION

Let c be the cardinality of collection $\mathfrak{C}$, k_j the cardinality of the support of fuzzy subset $\mathfrak{B}_j$ and g_{ij} the membership grade of X_i in $\mathfrak{B}_j$ then

using level 2 fuzzy set reduction we state : $\underline{A}(\mathfrak{B}_j) = \bigcup_{1\le i\le k_j} g_{ij}.\, \tilde{a}(X_i)$ (25)

or using membership grades

$$(\forall t \in [0,1]) : \mu_{\underline{A}(\mathfrak{B}_j)}(t) = \max_{1\le i\le k_j} [\, g_{ij}.\, \mu_{\tilde{a}(X_i)}(t) \,] \quad (26)$$

The hyperscale Ψ will , then, be defined by : $\Psi(\mathfrak{B}_j) = F(\underline{A}(\mathfrak{B}_j))$ (27)

2.2.3- m-FLOU SET-VALUED SOFAs

Taking once more the basic result into consideration, we can define for each object X belonging to $\mathfrak{B}$, an m-flou set-valued SOFA (m = 2)

$$\underset{\sim}{a}(X) = ([\mathbf{P^*}(X), \mathbf{O_*}(X)] , [\mathbf{P_*}(X), \mathbf{O^*}(X)]) \qquad (28)$$

Once given the different m-flou set-valued SOFAs of the various objects,the fuzzy evaluation of each fuzzy subset $\mathfrak{B}$ will be :

$$\underset{\sim}{A}(\mathfrak{B}) = ([\sum_{1\le i\le k} p_i \mathbf{P^*}(X_i), \sum_{1\le i\le k} p_i \mathbf{O_*}(X_i)],[\sum_{1\le i\le k} p_i \mathbf{P_*}(X_i), \sum_{1\le i\le k} p_i \mathbf{O^*}(X_i)]) \quad (29)$$

The result for each fuzzy subset $\mathfrak{B}$ is an m-flou set (with m = 2): $\underset{\sim}{A}(\mathfrak{B}) = ([L(\mathfrak{B}),U(\mathfrak{B})], [L'(\mathfrak{B}), U'(\mathfrak{B})])$ with $[L(\mathfrak{B}),U(\mathfrak{B})] \subseteq [L'(\mathfrak{B}), U'(\mathfrak{B})]$ The different fuzzy subsets could be ranked as follows :

$\mathfrak{B}' \ge \mathfrak{B} \Leftrightarrow [L(\mathfrak{B}),U(\mathfrak{B})] \le [L(\mathfrak{B}'),U(\mathfrak{B}')]$ and $[L'(\mathfrak{B}),U'(\mathfrak{B})] \le [L'(\mathfrak{B}'),U'(\mathfrak{B}')]$

where $\le$ is Ponsard's linear ordering defined on the set of intervals of [0,1]

DEFINITIONS 2.1

Let $\mathfrak{B}$, $\mathfrak{B}'$ and $\mathfrak{B}_i$ be fuzzy subsets belonging to $\mathfrak{C}$ and Ψ any hyperscale defined in the foregoing sections, then a hyperrelation of dominance $\mathbf{D}$ can be defined in $\mathfrak{C}$ by :

$$\mathfrak{B}' \ \mathbf{D} \ \mathfrak{B} \Leftrightarrow \Psi(\mathfrak{B}') \ge \Psi(\mathfrak{B}) \qquad (30)$$

Furthermore, we say that :

1°) $\mathfrak{B}$ is hyperdominant $\Leftrightarrow \forall \mathfrak{B}_i \in \mathfrak{C}, \mathfrak{B}_i \neq \mathfrak{B} : \Psi(\mathfrak{B}) > \Psi(\mathfrak{B}_i)$

2°) $\mathfrak{B}$ is hyperdominated $\Leftrightarrow \forall \mathfrak{B}_i \in \mathfrak{C}, \mathfrak{B}_i \neq \mathfrak{B} : \Psi(\mathfrak{B}) < \Psi(\mathfrak{B}_i)$

2.3 - INTER-SUBSETS OVERALL DISCREPANCY EVALUATION

The normalized possibilistic measure $\hat{U}$ of non-specificity proposed by Higashi and Klir will be employed to evaluate the inter-subsets overall discrepancy . Let $\Phi = \{ \Phi_1, \Phi_2, ..., \Phi_c \}$ be the appreciation set of the c fuzzy subsets corresponding to a given hyperscale and let $\pi(\Phi) = \{ \pi_1, \pi_2,...,\pi_c \}$, with $\pi_i > \pi_{i+1}$ for i=1,..., c and $\pi_{c+1} = 0$, be the set of normalized and decreasingly ordered evaluations of the c fuzzy subsets, in this case we state :

$$\hat{U}[\pi(\Phi)] = \frac{1}{\log_2(c)} \sum_{i=1}^{i=c} (\pi_i - \pi_{i+1}) \log_2(i) \qquad (31)$$

and if we define a function D by : $D(\Phi) = 1 - \hat{U}[\pi(\Phi)]$ (32)

then, given the properties of $\hat{U}$, D satisfies the following straightforward properties :

1) $0 \le D(\Phi) \le 1$;

2) D is invariant with respect to permutations of the evaluations ;

3) D does not change if we add null evaluations ,
4) If $\pi(\Phi) \leq \pi(\Phi')$, then $D(\Phi') \leq D(\Phi)$,
5) Maximum : $D(\Phi) = 1$, if all the evaluations are null except one,
6) Minimum : $D(\Phi) = 0$, if all the evaluations are equal .
Thus, D can be used as inter-subsets overall discrepancy indicator of the fuzzy subsets evaluations distribution .

CONCLUDING REMARK

It is possible to apply inter alia the methodology exposed in the present paper to interregional multidimensional discrepancy analysis if the data matrix is chosen to be a multiregional welfare matrix, the object sample set Ω is chosen to be a system of regions and $\mathbf{C}$ a collection of fuzzy clusters of regions obtained by means of any suitable Q-technique .

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