# INTERVAL-VALUE L-FUZZY SET AND ITS DECOMPOSITION THEOREM

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**Abstract**

In this paper, we proposed a new concept of interval-value L-fuzzy set and, defined its three basic operations. In addition, we presented so-called $[\lambda_1, \lambda_2]$-cutset and, discussed its basic properties. As the main results of this paper, we established a number of decomposition theorems for interval-value L-fuzzy sets.

**Keywords**: Interval-value L-fuzzy set, complete lattice, $[\lambda_1, \lambda_2]$-cutset, upper(lower)-L-fuzzy set, decomposition theorem for interval-value L-fuzzy sets.

## 1. Introduction

At present interval-value fuzzy sets (for fixed $x \in X$, $\mu(x)$ is a closed interval contained in the interval $[0,1]$) have been applied to some fields, such as controller, signal transmission, approximate inference, etc. (see[1-4]). In this paper, replacing $[0,1]$ by the general lattice L, we have presented the concept of interval-value L-fuzzy sets and, discussed their three basic operations. In addition, we have generalized the $\lambda$-cutset for fuzzy sets ($\lambda \in [0,1]$) to $[\lambda_1, \lambda_2]$-cutset for L-fuzzy sets ($[\lambda_1, \lambda_2]$ is a closed interval contained in the lattice L) and, discussed its main properties. Basing on them, we established many decomposition theorems for interval-value L-fuzzy sets. Each of them is a generalization for the decomposition theorem of fuzzy sets.

## 2. Preliminaries

Throughout this paper, X always denotes a crisp set and p(X) power set of X. In this paper, $(L, \leq, \vee, \wedge)$ always, unless specifically stated, denotes a complete lattice, and 0 and 1 its smallest and greatest elements respectively.

**Definition 2.1** Let $': L \to L$ be a map, we call it an order-reversing involution on L, if the following two conditions hold:

(1) $(a')' = a$, $a \in L$ ;

(2) If $a \leq b$ then $b' \leq a'$, $a, b \in L$.

We will denote by $L^X$ all L-fuzzy sets [5] on X and, by $\cup$, $\cap$, c their operations of union, intersection and complement respectively, where for $A \in L^X$, $A^c(x) = A(x)'$, $x \in X$.

**Definition 2.2** A lattice $(L, \leq)$ is called dense, if for arbitrary $\alpha$, $\beta \in L$ and $\alpha < \beta$ (i.e. $\alpha \leq \beta$ and $\alpha \neq \beta$), there exists some $\lambda \in L$ such that $\alpha < \lambda < \beta$.

It is not difficult to prove the following proposition.

**Proposition 2.1** Let L be a complete dense lattice. Then we have

$\beta = \vee \{\alpha \in L : \alpha < \beta\}$, $\beta = \wedge \{\alpha \in L : \beta < \alpha\}$.

## 3. Construction of I(L)

**Definition 3.1** Let $a, b \in L$ and $a \leq b$, then

$$[a,b] = \{\lambda \in L : a \leq \lambda \leq b\}$$

is called an (closed) interval.

We will denote by I(L) all intervals on L. It must be pointed out $L \subset I(L)$, because for any $a \in L$, $a = [a,a] \in I(L)$.

**Definition 3.2** For $[a_t, b_t] \in I(L)$, $t \in T$.

(1) $\bigvee_{t \in T} [a_t, b_t] = [\bigvee_{t \in T} a_t, \bigvee_{t \in T} b_t]$,

(2) $\bigwedge_{t \in T} [a_t, b_t] = [\bigwedge_{t \in T} a_t, \bigwedge_{t \in T} b_t]$,

(3) $[a_t, b_t]' = [b_t', a_t']$.

**Definition 3.3** For $[a_i, b_i] \in I(L)$, $i = 1, 2$.

(1) $[a_1, b_1] = [a_2, b_2]$ if $a_1 = a_2$ and $b_1 = b_2$,

(2) $[a_1, b_1] \leq [a_2, b_2]$ if $[a_1, b_1] \vee [a_2, b_2] = [a_2, b_2]$,

(3) $[a_1, b_1] < [a_2, b_2]$ if $[a_1, b_1] \leq [a_2, b_2]$ and $[a_1, b_1] \neq [a_2, b_2]$.

Without particular difficult we can prove the following proposition.

**Proposition 3.1** Let $a_i, b_i \in L$, $i=1,2$, and "'" as Definition 3.2(3). Then

(1) $[a_1, b_1] \leq [a_2, b_2]$ iff $a_1 \leq a_2$ and $b_1 \leq b_2$.

(2) "'" is an order-reversing involution on $I(L)$.

From Definition 3.2 we see that operations $\vee$, $\wedge$, $'$ on intervals are fully transformed into corresponding operations $\vee$, $\wedge$, $'$ on end points of these intervals, thus the operational rules in $(L, \leq, \vee, \wedge, ')$ are fully suitable for $(I(L), \leq, \vee, \wedge, ')$ and as a result we get

**Theorem 3.1** $(I(L), \leq, \vee, \wedge, ')$ is a complete lattice with an order-reversing involution " ' ", its smallest and greatest elements are $[0]=[0,0]$ and $[1]=[1,1]$ respectively.

## 4. Interval-value L-fuzzy set and its upper(lower)L-fuzzy sets

**Definition 4.1** We call a map

$$A: X \to I(L)$$

an interval-value L-fuzzy set (or IVL-fuzzy set). We wil denote by $I(L)^X$ all IVL-fuzzy sets on X.

**Remark 4.1** When $L=[0,1]$, an IVL-fuzzy set is an interval-value fuzzy set. If for any $x \in X$, $A(x) \in L \subset I(L)$, then an IVL-fuzzy set is the general L-fuzzy set.

**Definition 4.2** For $A_t \in I(L)^X$, $t \in T$.

(1) $(\bigcup_{t \in T} A_t)(x) = \bigvee_{t \in T} A_t(x)$.  (2) $(\bigcap_{t \in T} A_t)(x) = \bigwedge_{t \in T} A_t(x)$.

(3) $A_t^c(x) = (A_t(x))'$.  (4) $A_t = A_s$ if $A_t(x) = A_s(x)$, $s \in T$.

(5) $A_t \subset A_s$ if $A_t(x) \leq A_s(x)$, $s \in T$.

**Theorem 4.1** $(I(L)^X, \subset, \cup, \cap, c)$ is a complete lattice with an order-reversing involution "c", its smallest and greatest elements are $\underline{0}$ and $\underline{1}$ respectively, where

$\underline{0}(x)=[0]$, $\underline{1}(x)=[1]$.

Proof. Straightforward.

**Definition 4.3** For any $A \in I(L)^X$, suppose $A(x)=[a^-(x), a^+(x)]$, $x \in X$. We call L-fuzzy sets

$A^{+}: X \to L,\ A^{+}(x)=a^{+}(x),\ x \in X$

and

$A^{-}: X \to L\ ,\ A^{-}(x)=a^{-}(x)\ ,\ x \in X$

upper and lower-L-fuzzy set of A respectively.

**Theorem 4.2** Let $A_t \in I(L)^X$, $t \in T$, then

(1) $(\bigcup_{t\in T} A_t)^{-} = \bigcup_{t\in T} A_t^{-}$, $(\bigcup_{t\in T} A_t)^{+} = \bigcup_{t\in T} A_t^{+}$.

(2) $(\bigcap_{t\in T} A_t)^{-} = \bigcap_{t\in T} A_t^{-}$, $(\bigcap_{t\in T} A_t)^{+} = \bigcap_{t\in T} A_t^{+}$.

(3) $(A_t^{c})^{-} = (A_t^{+})^{c}$ , $(A_t^{c})^{+} = (A_t^{-})^{c}$ .

**Proof** . (1) For any $x \in X$, since

$$(\bigcup_{t\in T} A_t)(x) = \bigvee_{t\in T} A_t(x) = \bigvee_{t\in T} [A_t^{-}(x), A_t^{+}(x)]$$

$$=[\bigvee_{t\in T} A_t^{-}(x),\ \bigvee_{t\in T} A_t^{+}(x)] = [(\bigcup_{t\in T} A_t^{-})(x),\ (\bigcup_{t\in T} A_t^{+})(x)],$$

hence (1) holds.

The proof for (2) and (3) is similar to (1) .

## 5. $[\lambda_1, \lambda_2]$-cutsets

**Definition 5.1** Let $A \in I(L)^X$, $\lambda_1, \lambda_2 \in L$. We call orderly

$A_{[\lambda_1,\lambda_2]} = \{x \in X:\ \lambda_1 \leq A^{-}(x) \leq \lambda_2 \leq A^{+}(x)\}$, $\lambda_1 \leq \lambda_2$

$A_{(\lambda_1,\lambda_2]} = \{x \in X:\ \lambda_1 < A^{-}(x) \leq \lambda_2 \leq A^{+}(x)\}$, $\lambda_1 < \lambda_2$

$A_{[\lambda_1,\lambda_2)} = \{x \in X:\ \lambda_1 \leq A^{-}(x) \leq \lambda_2 < A^{+}(x)\}$, $\lambda_1 < \lambda_2$

$A_{(\lambda_1,\lambda_2)} = \{x \in X:\ \lambda_1 < A^{-}(x) \leq \lambda_2 < A^{+}(x)\}$, $\lambda_1 < \lambda_2$

$[\lambda_1, \lambda_2]$-cutset, $(\lambda_1, \lambda_2]$-cutset, $[\lambda_1, \lambda_2)$-cutset and $(\lambda_1, \lambda_2)$-cutset of A.

Now we will discuss the basic properties for cutsets. First recall two signs. For any $A \in L^X$ and $\alpha \in L$,

$A_{\alpha} = \{x \in X: A(x) \geq \alpha\}$, $A_{\underset{\cdot}{\alpha}} = \{x \in X:\ A(x) > \alpha\}$.

**Theorem 5.1** Let $A, B \in I(L)^X$. Then

(1) $(A \cup B)_{[\lambda_1,\lambda_2]} \supset A_{[\lambda_1,\lambda_2]} \cup B_{[\lambda_1,\lambda_2]} \cup (A^{-}_{\lambda_1} \cap B^{+}_{\lambda_2} \cap A^{-c}_{\lambda'_2}) \cup (B^{-}_{\lambda_1} \cap A^{+}_{\lambda_2} \cap B^{-c}_{\lambda'_2})$.

(2) $(A \cup B)_{(\lambda_1,\lambda_2)} \supset A_{(\lambda_1,\lambda_2)} \cup B_{(\lambda_1,\lambda_2)} \cup (A^{-}_{\underset{\cdot}{\lambda_1}} \cap B^{+}_{\underset{\cdot}{\lambda_2}} \cap A^{-c}_{\lambda'_2}) \cup (B^{-}_{\underset{\cdot}{\lambda_1}} \cap A^{+}_{\underset{\cdot}{\lambda_2}} \cap B^{-c}_{\lambda'_2})$.

(3) $(A \cap B)_{[\lambda_1,\lambda_2]} = A_{[\lambda_1,\lambda_2]} \cap B_{[\lambda_1,\lambda_2]} \cap A^{-}_{\lambda_1} \cap B^{+}_{\lambda_2} \cap A^{-c}_{\lambda'_2} \cap B^{-}_{\lambda_1} \cap A^{+}_{\lambda_2} \cap B^{-c}_{\lambda'_2}$.

(4) $(A \cap B)_{(\lambda_1,\lambda_2)} \subset A_{(\lambda_1,\lambda_2)} \cap B_{(\lambda_1,\lambda_2)} \cap A^{-}_{\underset{\cdot}{\lambda_1}} \cap B^{+}_{\underset{\cdot}{\lambda_2}} \cap A^{-c}_{\lambda'_2} \cap B^{-}_{\underset{\cdot}{\lambda_1}} \cap A^{+}_{\underset{\cdot}{\lambda_2}} \cap B^{-c}_{\lambda'_2}$.

Proof. We prove only (1).

We denote by E the right of (1). For any $x\in X$, we have

$$x\in E \Longrightarrow x\in A_{[\lambda_1,\lambda_2]} \text{ or } x\in B_{[\lambda_1,\lambda_2]} \text{ or } x\in A^-_{\lambda_1}\cap B^+_{\lambda_2}\cap A^{-c}_{\lambda_2}$$
$$\text{or } x\in B^-_{\lambda_1}\cap B^+_{\lambda_2}\cap B^{-c}_{\lambda_2}$$
$$\Longrightarrow \lambda_1< A^-(x)< \lambda_2< A^+(x) \quad \text{or} \quad \lambda_1< B^-(x)< \lambda_2< B^+(x)$$
$$\text{or} \quad \lambda_1< A^-(x)< \lambda_2< B^+(x) \quad \text{or} \quad \lambda_1< B^-(x)< \lambda_2< A^+(x)$$
$$\Longrightarrow \lambda_1< A^-(x)\vee B^-(x)< \lambda_2< A^+(x)\vee B^+(x)$$
$$\Longrightarrow \lambda_1< (A\cup B)^-(x)< \lambda_2< (A\cup B)^+(x) \Longrightarrow x\in (A\cup B)_{[\lambda_1,\lambda_2]}.$$

Hence (1) holds. The proof (2) − (4) is similar to (1).

**Theorem 5.2** For any $A\in I(L)^X$, we have

$$A_{(\lambda_1,\lambda_2)}\subset A_{[\lambda_1,\lambda_2)}\subset A_{[\lambda_1,\lambda_2]}, \quad A_{(\lambda_1,\lambda_2)}\subset A_{(\lambda_1,\lambda_2]}\subset A_{[\lambda_1,\lambda_2]}.$$

proof. It is clear.

**Theorem 5.3** Let $A\in I(L)^X$, $[\lambda_{1t},\lambda_{2t}]\in I(L)$, $t\in T$. Then

$$\bigcap_{t\in T} A_{[\lambda_{1t},\lambda_{2t}]} = A_{\bigvee_{t\in T}[\lambda_{1t},\lambda_{2t}]} \quad . \tag{5.1}$$

Proof. For any $x\in X$, since

$$x\in\bigcap_{t\in T} A_{[\lambda_{1t},\lambda_{2t}]} \Longleftrightarrow \forall t\in T,\ x\in A_{[\lambda_{1t},\lambda_{2t}]}$$
$$\Longleftrightarrow \forall t\in T,\ \lambda_{1t}< A^-(x)< \lambda_{2t}< A^+(x)$$
$$\Longleftrightarrow \bigvee_{t\in T}\lambda_{1t}< A^-(x)< \bigvee_{t\in T}\lambda_{2t}< A^+(x)$$
$$\Longleftrightarrow x\in A_{[\bigvee_{t\in T}\lambda_{1t},\bigvee_{t\in T}\lambda_{2t}]} \Longleftrightarrow x\in A_{\bigvee_{t\in T}[\lambda_{1t},\lambda_{2t}]},$$

hence (5.1) holds.

## 6. Decomposition theorems

**Definition 6.1** Let $A\in I(L)^X$, $[\lambda_1,\lambda_2]\in I(L)$. We provide $[\lambda_1,\lambda_2]A \in I(L)^X$ by

$$([\lambda_1,\lambda_2]A)(x)=[\lambda_1,\lambda_2]\wedge A(x).$$

Specifically, if $A\in P(X)$, then

$$([\lambda_1,\lambda_2]A)(x)=[\lambda_1,\lambda_2]\wedge [\chi_A(x),\chi_A(x)],$$

where

$$\chi_A(x)= \begin{cases} 1, & x\in A, \\ 0, & x\bar{\in} A, \end{cases} \quad 1,0\in L.$$

**Proposition 6.1** Let $[\lambda_1,\lambda_2]$, $[\lambda_3,\lambda_4]\in I(L)$, $A,B\in I(L)^X$. Then

(1) $[\lambda_1,\lambda_2]A\subset[\lambda_3,\lambda_4]A$ if $[\lambda_1,\lambda_2]<[\lambda_3,\lambda_4]$.

(2) $[\lambda_1,\lambda_2]A \subset [\lambda_1,\lambda_2]B$ if $A \subset B$.

Proof. It is easy.

**Decomposition Theorem I** For any $A \in I(L)^X$, we have

$$A = \bigcup_{[\lambda_1,\lambda_2]\in I(L)} [\lambda_1,\lambda_2]A_{[\lambda_1,\lambda_2]}$$

Proof. For a arbitrary $x \in X$, from

$$\chi_{A_{[\lambda_1,\lambda_2]}}(x) = \begin{cases} 1, & \lambda_1 \leq A^-(x) \leq \lambda_2 \leq A^+(x), \\ 0, & \text{otherwise,} \end{cases}$$

we get

$$\Big(\bigcup_{[\lambda_1,\lambda_2]\in I(L)} [\lambda_1,\lambda_2]A_{[\lambda_1,\lambda_2]}\Big)(x) = \bigvee_{[\lambda_1,\lambda_2]\in I(L)} ([\lambda_1,\lambda_2] \wedge [\chi_{A_{[\lambda_1,\lambda_2]}}(x), \chi_{A_{[\lambda_1,\lambda_2]}}(x)])$$

$$= \bigvee \{[\lambda_1,\lambda_2]: \lambda_1 \leq A^-(x) \leq \lambda_2 \leq A^+(x)\} = [A^-(x), A^+(x)] = A(x)$$

**Corollary 6.1** For $A,B \in I(L)^X$, we have

$A \subset B$ iff $A_{[\lambda_1,\lambda_2]} \subset B_{[\lambda_1,\lambda_2]}$, $\forall [\lambda ,\lambda ] \in I(L)$;

$A = B$ iff $A_{[\lambda_1,\lambda_2]} = B_{[\lambda_1,\lambda_2]}$, $\forall [\lambda ,\lambda ] \in I(L)$.

**Decomposition Theorem II** Let L be a dense complete lattice and $A \in I(L)^X$, then

(1) $A = \bigcup_{\substack{\lambda_1<\lambda_2 \\ \lambda_1,\lambda_2\in L}} [\lambda_1,\lambda_2]A_{(\lambda_1,\lambda_2)}$; (2) $A = \bigcup_{[\lambda_1,\lambda_2]\in I(L)} [\lambda_1,\lambda_2]A_{[\lambda_1,\lambda_2)}$;

(3) $A = \bigcup_{[\lambda_1,\lambda_2]\in I(L)} [\lambda_1,\lambda_2]A_{(\lambda_1,\lambda_2]}$.

Proof. To prove only (1).

$\forall x \in X$, we have

$$\Big(\bigcup_{\substack{\lambda_1<\lambda_2 \\ \lambda_1,\lambda_2\in L}} [\lambda_1,\lambda_2]A_{(\lambda_1,\lambda_2)}\Big)(x) = \bigvee_{\substack{\lambda_1<\lambda_2 \\ \lambda_1,\lambda_2\in L}} ([\lambda_1,\lambda_2] \wedge [\chi_{A_{(\lambda_1,\lambda_2)}}(x), \chi_{A_{(\lambda_1,\lambda_2)}}(x)])$$

$$= \bigvee \{[\lambda_1,\lambda_2]: \lambda_1 < A^-(x) \leq \lambda_2 < A^+(x)\} = [\bigvee_{A^-(x)>\lambda_1} \lambda_1, \bigvee_{A^+(x)>\lambda_2} \lambda_2]$$

$$= [A^-(x), A^+(x)] = A(x).$$

Hence (1) holds.

**Decomposition Theorem III** Let L be a dense complete lattice, $A \in I(L)^X$. If the map

$$H: I(L) \to P(x) \quad [\lambda_1,\lambda_2] \mapsto H(\lambda_1,\lambda_2)$$

satisfies the following condition

$$A_{(\lambda_1,\lambda_2)} \subset H(\lambda_1,\lambda_2) \subset A_{[\lambda_1,\lambda_2]},$$

then

$$A = \bigcup_{[\lambda_1,\lambda_2]\in I(L)} [\lambda_1,\lambda_2]H(\lambda_1,\lambda_2). \qquad (6.1)$$

**Proof**. For any $[\lambda_1,\lambda_2]\in I(L)$, from Proposition 6.1 we have

$$[\lambda_1,\lambda_2]A_{(\lambda_1,\lambda_2)} \subset [\lambda_1,\lambda_2]H(\lambda_1,\lambda_2) \subset [\lambda_1,\lambda_2]A_{[\lambda_1,\lambda_2]}.$$

It follows from Decomposition Theorem II (1) that

$$A = \bigcup_{\substack{\lambda_1<\lambda_2\\ \lambda_1,\lambda_2\in L}} [\lambda_1,\lambda_2]A_{(\lambda_1,\lambda_2)} \subset \bigcup_{0\leq\lambda_1\leq\lambda_2\leq 1} [\lambda_1,\lambda_2]H(\lambda_1,\lambda_2) \subset \bigcup_{0\leq\lambda_1\leq\lambda_2\leq 1} [\lambda_1,\lambda_2]A_{[\lambda_1,\lambda_2]}$$

$=A$.

So (6.1) holds.

**Definition 6.2** Let $\lambda\in L, A, B\in P(x), A\subset B$. We define $\lambda[A,B]\in I(L)^X$ by

$$(\lambda[A,B])(x)=[\lambda\wedge\chi_A(x), \lambda\wedge\chi_B(x)].$$

**Decomposition Theorem IV** For any $A\in I(L)^X$, we have

$$A=\bigcup_{\lambda\in L}\lambda[A^-_\lambda, A^+_\lambda].$$

**Proof**. Using the decomposition theorem of the general L-fuzzy sets, for any $x\in X$ we have

$$A^-(x)=\bigvee_{\lambda\in L}(\lambda\wedge\chi_{A^-_\lambda}(x)), \qquad A^+(x)=\bigvee_{\lambda\in L}(\lambda\wedge\chi_{A^+_\lambda}(x)).$$

It follows that

$$(\bigcup_{\lambda\in L}\lambda[A^-_\lambda, A^+_\lambda])(x)=\bigvee_{\lambda\in L}(\lambda[A^-_\lambda, A^+_\lambda])(x)=\bigvee_{\lambda\in L}[\lambda\wedge\chi_{A^-_\lambda}(x), \lambda\wedge\chi_{A^+_\lambda}(x)]$$

$$=[\bigvee_{\lambda\in L}(\lambda\wedge\chi_{A^-_\lambda}(x)), \bigvee_{\lambda\in L}(\lambda\wedge\chi_{A^+_\lambda}(x))]=[A^-(x), A^+(x)]=A(x).$$

**Remark 6.1** Decomposition Theorem IV shows that the decomposition of IVL-fuzzy set can be got by decomposing its upper and lower-L-fuzzy sets.

Corollary 6.2 Let L be a dense complete lattice, $A\in I(L)^X$, then

(1) $A=\bigcup_{\lambda\in L}\lambda[A^-_{\underset{\cdot}{\lambda}}, A^+_{\underset{\cdot}{\lambda}}]$. (2) $A=\bigcup_{\lambda\in L}\lambda[A^-_{\underset{\cdot}{\lambda}}, A^+_{\lambda}]$.

(3) $A=\bigcup_{\lambda\in L}\lambda[A^-_{\lambda}, A^+_{\underset{\cdot}{\lambda}}]$.

**Decomposition Theorem V** Let L be a dense complete lattice, $A\in I(L)^X$. If the maps

$H_i: L\to P(x) \quad \lambda\mapsto H_i(\lambda) \quad i=1,2$

satisfy the following conditions

$$A^-_{\underset{\cdot}{\lambda}}\subset H_1(\lambda)\subset A^-_\lambda, \quad A^+_{\underset{\cdot}{\lambda}}\subset H_2(\lambda)\subset A^+_\lambda,$$

then

$$A = \bigcup_{\lambda \in L} \lambda\, [H_1(\lambda), H_2(\lambda)] \tag{6.2}$$

Proof. It is easy to see that

$$\lambda\, [A_\lambda^-, A_\lambda^+] \subset \lambda\, [H_1(\lambda), H_2(\lambda)] \subset \lambda\, [A_\lambda^-, A_\lambda^+],\ \lambda \in L.$$

By Decomposition Theorem IV and Corollary 6.2 (i), we get

$$A = \bigcup_{\lambda \in L} \lambda\, [A_\lambda^-, A_\lambda^+] \subset \bigcup_{\lambda \in L} \lambda\, [H_1(\lambda), H_2(\lambda)] \subset \bigcup_{\lambda \in L} \lambda\, [A_\lambda^-, A_\lambda^+] = A.$$

Thus (6.2) holds.

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