FUZZY LOGIC MODEL OF SINGLE NEURON

J.B. Kiszka*, and M.M. Gupta**

***Electrical Engineering**
Lakehead University
Thunder Bay, Ontario
P7B 5E1 Canada

****College of Engineering**
University of Saskatchewan
Saskatoon, Sask.
S7N 0W0 Canada

Abstract: A fuzzy logic model of a single neuron is proposed. Multivariable architecture of fuzzy neuron is incorporated.

1. Introduction.

Observing biological mechanisms of a human being intelligence, one may conclude that a single neuron may be viewed as a more complex processing unit than being a single integrating device.

We impose a hypothesis that a neural processing phenomena is inherently uncertain.

To shape a vague nature of a neuron we use fuzzy logic theory as a formal tool to express the uncertain essence of neuron systems.

2. Fuzzy logic static model.

Let us assume a single neuron with 3 inputs and 1 output as in Figure 1.

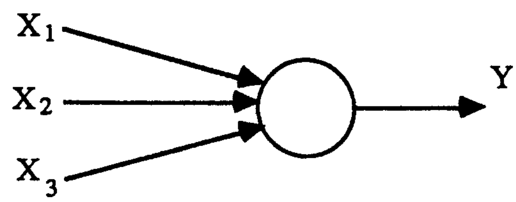

Figure 1. Single fuzzy neuron

Suppose that inputs and outputs signals exhibits fuzzy nature. Therefore, the linguistic description of the neuron could be expressed as [1]

$$\{\text{IF } X_{1(i)} \text{ AND } X_{2(i)} \text{ AND } X_{3(i)} \text{ THEN } Y_{(i)}, \text{ ALSO}\}, i = 1, 2, 3, \ldots I \quad (1)$$

where $X_{K(i)}$ is the fuzzy value of the k-input variable, and $Y_{(i)}$ is the fuzzy value of the output variable, and $i = 1, 2, 3, \ldots, I$ is the number of rules.

The single fuzzy logic neuron can be now stated as [1]

$$Y = X_1 \circ R_1 \,\Delta\, X_2 \circ R_2 \,\Delta\, X_3 \circ R_3 \quad (2)$$

where $Y, X_1, X_2,$ and X_3 are present output and current inputs, respectively;

o stands for composition operator; Δ means aggregation operator.

Fuzzy relations R_1, R_2, and R_3 are defined as

$$R_1 = \bigvee_i \{X_{1(i)} \wedge Y_{(i)}\},\ R_2 = \bigvee_i \{X_{2(i)} \wedge Y_{(i)}\},\ R_3 = \bigvee_i \{X_{3(i)} \wedge Y_{(i)}\}.$$

where $\vee$ stands for max operators, and $\wedge$ stands for min operator.

3. Fuzzy Logic dynamic model.

To express a dynamic nature of a single neuron let us impost a past experience of the neuron X_n; and an intended experience X_{n+1}, where n is a discrete-time variable. Assuming discrete-time fuzzy stimuli U_n^k and an output Y_n, a fuzzy logic neuron could be depicted as in Fig. 2

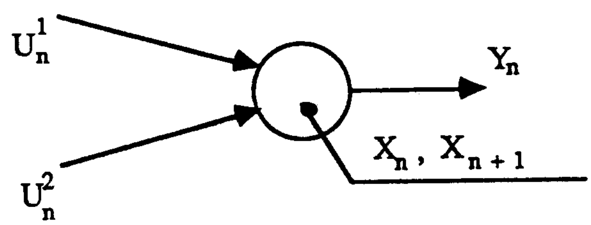

Fig. 2 Two-input fuzzy dynamic neuron.

A behavior of the neuron with intended and past experience as a response to fuzzy stimuli can be described by

$$\left\{ \text{IF } X_{n(i)} \text{ AND } U^1_{n(i)} \text{ AND } U^2_{n(i)} \text{ THEN } X_{n+1\,(i)} \text{ AND } Y_{n(i)}, \text{ ALSO} \right\} \quad (3)$$

where $i = 1, 2, 3, \dots, I$ and $n = 0, 1, 2, 3, \dots$

The dynamic behavior of the neuron can be expressed as

$$\begin{cases} X_{n+1} = X_n \circ R_1 \,\Delta\, U_n^1 \circ R_2 \,\Delta\, U_n^2 \circ R_3 \\ Y_n = X_n \circ R_4 \,\Delta\, U_n^1 \circ R_5 \,\Delta\, U_n^2 \circ R_6 \end{cases} \tag{4}$$

Let us calculate a response of the neuron for a sequence of stimuli

U_n^1 , U_n^2 , $n = 0, 1,2, \ldots , N$.

For $n = 0$ we may compute X_1 using Equation (4)

$$X_1 = X_0 \circ R_1 \,\Delta\, U_0^1 \circ R_2 \,\Delta\, U_0^2 \circ R_3 \tag{5}$$

where X_0 is an initial experience, and U_0^1 and U_0^2 initial stimuli.

For $n = 1$, and using (5) we get from Equation (4)

$$X_2 = X_1 \circ R_1 \,\Delta\, U_1^1 \circ R_2 \,\Delta\, U_1^2 \circ R_3 \tag{6}$$

and consequantly

$$X_2 = X_0 \circ R_1 \circ R_2 \,\Delta\, U_0^1 \circ R_2 \circ R_1 \,\Delta\, U_1^1 \circ R_2 \,\Delta\, U_0^2 \circ R_3 \circ R_1 \,\Delta\, U_1^2 \circ R_3 \tag{7}$$

In general we get

$$X_n = X_0 \circ R_1^n \,\Delta \bigwedge_{n-1}^{i=0} U_i^1 \circ R_2 \circ R_1^{n-i-1} \,\Delta \bigwedge_{n-1}^{i=0} U_i^2 \circ R_3 \circ R_1^{n-1-i} \tag{8}$$

where R_1^n means n-times composition of fuzzy relation R_1, i.e.
$R_1^n = R_1 \circ R_1 \circ \ldots \circ R_1$ n-times.

For a discrete time $n = k$ a current experience of the neuron could be stated as

$$X_k = X_0 \circ R_1^k \, \Delta \bigwedge_{i=1}^{k-1} U_i^1 \circ R_2 \circ R_1^{k-i-1} \, \Delta \bigwedge_{i=1}^{k-1} U_i^2 \circ R_3 \circ R_1^{k-i-1} \tag{9}$$

$k = 1, 2, 3, 4, \ldots$

In view of Equations (4) and (9) a current output of the neuron is expressed as

$$Y_k = X_0 \circ R_1^k \circ R_4 \, \Delta \left[\bigwedge_{i=1}^{k-1} \left[U_i^1 \circ R_2 \circ R_1^{k-i-1} \circ R_4 \right] \Delta \, U_k^1 \circ R_5 \right] \Delta \left[\bigwedge_{i=1}^{k-1} \left[U_i^2 \circ R_3 \circ R_1^{k-i-1} \circ R_4 \right] \Delta \, U_k^2 \circ R_6 \tag{10}$$

$k = 1, 2, 3, \ldots$

Considering the current experience of the neuron (9) it is worth to note that the following terms contribute to X_k:

the initial experience term $X_0 \circ R_1^k$; the sequence input terms $\bigwedge_{i=1}^{k-1} U_i^1 \circ R_2 \circ R_1^{k-i-1}$ and $\bigwedge_{i=1}^{k-1} U_i^2 \circ R_3 \circ R_1^{k-i-1}$, respectively.

4. Summary.

An unified theory of a neuron based fuzzy logic has been proposed.

References.

[1] M.M. Gupta, J.B. Kiszka, and G.M. Trojan, Multivariable structure of fuzzy control systems, IEEE Trans., SMC-16, No. 5, Sept./Oct. 1986.