F-TRUTH OF (T,⊥) FUZZY LOGICAL FORMULAE

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ABSTRACT

In this paper,the F-truth of (T,⊥) fuzzy logical formulae and the relations between it and the (^,v) Boolean tautology are discussed,thus, we transfer the infinite problem of the F-truth of (T,⊥) fuzzy logical formulae into the finite problem of the truth of (^,v) Boolean logical formulae. Thereby, we create favourable conditions for further studying the simplification of (T,⊥) fuzzy logical formulae.

KEYWORDS:(T,⊥) Fuzzy Logical Formula,T(S)-norm,F-true(false), Boolean Tautology (Inconsistent),Normal Form.

1.INTRODUCTION

The fuzzy logical formulae which are constituted by the logical operator pair (^,v) have been discussed in some papers [1-6],but we often feel that the logical operator pair (^,v) is too rough to satisfy need of the logical inference and logical circuit diadram design in application,therefor,it is necessary to discuss fuzzy logical formulae which are constituted by more wide logical operator pair.We have discussed the generalized fuzzy logical formulae [7] which are constituted by T-norm T and S-norm ⊥ [8,9].In this paper,our aim is discussion on the F-truth of (T,⊥) fuzzy logical formulae which are constituted by T-norm T and S-norm ⊥ .It is necessary to simplify (T,⊥) fuzzy logical formulae.

Let variable set be $\{x_1,\dots,x_n\}$.The domain of definition of x_i is the closed interval [0,1] (i=1,...,n).

DEFINITION 1.The fuzzy variable x or its complementary $\bar{x}$ is called literal.In the fuzzy logical formula F [1,2,6], if we have T-norm T instead of ^, S-norm ⊥ instead of v, then the fuzzy logical formula F is called a (T,⊥) fuzzy logical formula,write it as (T,⊥)F.The following fuzzy logical formula is called a (T)phrase

$$Y_1 T Y_2 T \cdots T Y_m \qquad (1)$$

write it as

$$(T) Y_1 Y_2 \cdots Y_m \qquad (2)$$

The (T)phrase without complementary pair is called a (T)single term, the (T)phrase which contains complementary pair is called a (T)complementary term.The following form is called a (⊥)sentence

$$Y_1 \perp Y_2 \perp \cdots \perp Y_m \qquad (3)$$

write it as

$$(\perp) Y_1 Y_2 \cdots Y_m \qquad (4)$$

Let

$$y_1 y_2 \cdots y_m = p$$

then (2) is written as

$$(T)p \qquad (5)$$

and (4) is written as

$$(\perp)p \qquad (6)$$

Where, $y_i \in \{x_1, \ldots, x_n, \bar{x}_1, \ldots, \bar{x}_n\}$.

We denote '(T)single term', '(T)complementary term' and 'complementary pair' by '(T)ST', '(T)CT' and 'CP' respectively.

DEFINITION 2. Let

$$(\perp)p = (\perp)y_1 y_2 \cdots y_m \qquad (7)$$

$$(T)p = (T)y_1 y_2 \cdots y_m \qquad (8)$$

$\forall A = (a_1, \ldots, a_n) \in [0,1]^n$,

$$((\perp)p)(A) = y_1(A) \perp y_2(A) \perp \cdots \perp y_m(A) \qquad (9)$$

$$((T)p)(A) = y_1(A) T y_2(A) T \cdots T y_m(A) \qquad (10)$$

where,

$$y_j(A) = \begin{cases} a_i, & y_j = x_i, \\ N(a_i), & y_j = \bar{x}_i. \end{cases}$$

$j = 1, \ldots, m$, and N is a pseudo-complement mapping [8].

DEFINITION 3. The formula

$$(T,\perp)F = ((T)p_1) \perp ((T)p_2) \perp \cdots \perp ((T)p_m) \qquad (11)$$

is called a $(T\text{-}\perp)$ normal form. $\forall A \in [0,1]^n$,

$$((T,\perp)F)(A) = ((T)p_1)(A) \perp ((T)p_2)(A) \perp \cdots \perp ((T)p_m)(A).$$

The formula

$$(T,\perp)F = ((\perp)p_1) T ((\perp)p_2) T \cdots T ((\perp)p_m) \qquad (12)$$

is called a $(\perp\text{-}T)$ normal form. $\forall A \in [0,1]^n$,

$$((T,\perp)F)(A) = ((\perp)p_1)(A) T ((\perp)p_2)(A) T \cdots T ((\perp)p_m)(A).$$

DEFINITION 4. Let $(T,\perp)F_1$, $(T,\perp)F_2$ be $(T,\perp)$ fuzzy logical formulae. If $\forall A \in [0,1]^n$,

$$((T,\perp)F_1)(A) \geqslant ((T,\perp)F_2)(A) \qquad (13)$$

then, write it as

$$(T,\perp)F_1 \geqslant (T,\perp)F_2$$

DEFINITION 5. Let $(T,\perp)F$ be a $(T,\perp)$ fuzzy logical formula. It is called F-true if $\forall A \in [0,1]^n$,

$$((T,\perp)F)(A) \geqslant 0.5 \qquad (14)$$

It is called F-false if $\forall A \in [0,1]^n$,

$$((T,\perp)F)(A) \leqslant 0.5 \qquad (15)$$

DEFINITION 6. T-norm T is called regular if $\forall a,b \in (0,1]$,

$$a \, T \, b > 0 \qquad (16)$$

S-norm $\perp$ is called regular if $\forall a,b \in [0,1)$,

$$a \perp b < 1 \qquad (17)$$

2. F-TRUTH OF $(T,\perp)$ FUZZY LOGICAL FORMULAE

THEOREM 1.

(a). Let

$$(T)p = y_1 T y_2 T \cdots T y_m$$

be a (T)phrase. Then

(T)p is (T)CT iff (T)p is F-false.

(b). Let

$$(\perp)p = y_1 \perp y_2 \perp \cdots \perp y_m$$

be a $(\perp)$sentence. Then

$(\perp)p$ contains CP iff $(\perp)p$ is F-true.

Proof.(a).($\Longrightarrow$) Let $y_i = \bar{y}_j$.

$$\forall A\in[0,1]^n, (y_1 T\ldots T y_m)(A) = y_1(A)T\ldots Ty_m(A) \leqslant y_i(A)Ty_j(A)$$
$$= \bar{y}_j(A)Ty_j(A) \leqslant \bar{y}_j(A)\wedge y_j(A) \leqslant 0.5$$

($\Longleftarrow$) If $(T)p$ is not $(T)CT$, then $\exists\ A\in[0,1]^n$, such that $((T)p)(A) = 1$, contradicting that $(T)p$ is F-false.

Dualistically, we have (b). Q.E.D.

THEOREM 2.

(a). Let

$$(T,\perp)F = ((T)p_1)\perp\ldots\perp((T)p_m)$$

be a $(T\text{-}\perp)$ normal form. Then,

$(T,\perp)F$ is F-false $\Longleftrightarrow$ every $(T)p_i$ is F-false $(i=1,\ldots,m)$.

$\exists\ (T)p_i$, $(T)p_i$ is F-true $\Longrightarrow$ $(T,\perp)F$ is F-true.

(b). Let

$$(T,\perp)F = ((\perp)p_1)T\ldots T((\perp)p_m)$$

be a $(\perp\text{-}T)$ normal form. Then,

$(T,\perp)F$ is F-true $\Longrightarrow$ every $(\perp)p_i$ is F-true $(i=1,\ldots,m)$.

$\exists\ (\perp)p_i$, $(\perp)p_i$ is F-false $\Longrightarrow$ $(T,\perp)F$ is F-false.

The proof is easy and hence omitted.

LEMMA 1.

(a). In fuzzy logic,

$$((T)p_1)\vee\ldots\vee((T)p_m) \leqslant ((T)p_1)\perp\ldots\perp((T)p_m)$$
$$\leqslant ((\wedge)p_1)\perp\ldots\perp((\wedge)p_m) \quad (18)$$
$$((\perp)p_1)\wedge\ldots\wedge((\perp)p_m) \geqslant ((\perp)p_1)T\ldots T((\perp)p_m)$$
$$\geqslant ((\vee)p_1)T\ldots T((\vee)p_m) \quad (19)$$

(b). In Boolean logic,

$$((T)p_1)\vee\ldots\vee((T)p_m) = ((\wedge)p_1)\perp\ldots\perp((\wedge)p_m)$$
$$= ((T)p_1)\perp\ldots\perp((T)p_m) = ((\wedge)p_1)\vee\ldots\vee((\wedge)p_m) \quad (20)$$
$$((\perp)p_1)\wedge\ldots\wedge((\perp)p_m) = ((\vee)p_1)T\ldots T((\vee)p_m)$$
$$= ((\perp)p_1)T\ldots T((\perp)p_m) = ((\vee)p_1)\wedge\ldots\wedge((\vee)p_m) \quad (21)$$

Proof.(a) is immediate from $T\leqslant\wedge\leqslant\vee\leqslant\perp$ (Theorem 6.2.1 in [6])

(b). Since $\forall a,b\in\{0,1\}$, $aTb = a\wedge b$ and $a\perp b = a\vee b$, we have the Eq. (20) and the Eq. (21). Q.E.D.

THEOREM 3. Let $(T,\perp)F$ be a $(T\text{-}\perp)$ normal form or $(\perp\text{-}T)$ normal form. Then

$(T,\vee)F$ is F-true $\Longrightarrow$ $(T,\perp)F$ is F-true

$\Longrightarrow$ $(T,\perp)F$ is Boolean tautology.

$(\wedge,\perp)F$ is F-false $\Longrightarrow$ $(T,\perp)F$ is F-false

$\Longrightarrow$ $(T,\perp)F$ is Boolean inconsistent.

Proof. This is immediate from Lemma 1. Q.E.D.

THEOREM 4.

(a). $((T)p_1)\perp\ldots\perp((T)p_m)$ is Boolean tautology iff $((\wedge)p_1)\vee\ldots\vee((\wedge)p_m)$ is F-true.

(b). $((T)p_1)\perp\ldots\perp((T)p_m)$ is Boolean inconsistent iff $((\wedge)p_1)\vee\ldots\vee((\wedge)p_m)$ is F-false.

(c). $((\perp)p_1)T\ldots T((\perp)p_m)$ is Boolean tautology iff $((\vee)p_1)\wedge\ldots\wedge((\vee)p_m)$ is F-true.

(d). $((\perp)p_1)T\ldots T((\perp)p_m)$ is Boolean inconsistent iff $((\vee)p_1)\wedge\ldots\wedge((\vee)p_m)$ is F-false.

Proof. The proof is immediate from Lemma 1(b) and Theorem 5.2.3 of [6]. Q.E.D.

THEOREM 5. Let $(T,\perp)F$ be a $(T\text{-}\perp)$ normal form or $(\perp\text{-}T)$ normal form. Then,

(a). $(\wedge,\perp)F$ is Boolean tautology $\Longrightarrow$ $(\wedge,\perp)F$ is F-true.

(b). $(T,\vee)F$ is Boolean inconsistent $\Longrightarrow$ $(T,\vee)F$ is F-false.

Proof. We prove only (a), similarly, we can prove (b). If $(\wedge,\perp)F$ is Boolean tautology, then it follows by Theorem 4 that $(\wedge,\vee)F$ is F-true. So, $(\wedge,\perp)F$ is F-true by Theorem 3. Q.E.D.

By Theorem 4 and Theorem 5, we have

COROLLARY 1. Let $(T,\perp)F$ be a $(T\text{-}\perp)$ normal form or a $(\perp\text{-}T)$ normal form. Then,

(a). $(\wedge,\perp)F$ is F-true iff $(\wedge,\perp)F$ is Boolean tautology.

(b). $(T,\vee)F$ is F-false iff $(T,\vee)F$ is Boolean inconsistent.

COROLLARY 2. Let $(T,\perp)F$ be a $(T\text{-}\perp)$ normal form. Then, $((\wedge)p_1)\perp\ldots\perp((\wedge)p_m)$ is F-true iff $((\wedge)p_1)\vee\ldots\vee((\wedge)p_m)$ is Boolean tautology iff $\forall (y_1,\ldots,y_m)\in L_1 x\ldots x L_m$, $\exists\, y_i, y_j, y_i = \bar{y}_j$ iff $((\wedge)p_{i_1})\vee\ldots\vee((\wedge)p_{i_k})$ is Boolean tautology. Where $(\wedge)p_{i_1},\ldots,(\wedge)p_{i_k}$ are (T)STs which do not include each other in $(\wedge)p_1,\ldots,(\wedge)p_m$. iff $\forall (y_1,\ldots,y_k)\in L_{i_1} x\ldots x L_{i_k}$, $\exists\, y_i, y_j, y_i = \bar{y}_j$. Where L_t denotes the set of the literals in $(\wedge)p_t$.

Proof. It follows by Corollary 1(a) and Lemma 1(b) that $((\wedge)p_1)\perp\ldots\perp((\wedge)p_m)$ is F-true iff $((\wedge)p_1)\vee\ldots\vee((\wedge)p_m)$ is Boolean tautology. As for the others equivalence see Lemma 1 of [3]. Q.E.D.

3. CONCLUSION

This paper we have set up some relations between the F-truth of $(T,\perp)$ fuzzy logical formulae and the truth of $(\wedge,\vee)$ Boolean logical formulae. It will play an important role in the minimization of $(T,\perp)$ fuzzy logical formula [10], for it transfers an infinite problem into a finite problem.

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