GENERALIZATION OF THE SPECIAL FUZZY NUMBER

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I. Generalization Of The Interval Grey Number.

In the light of the interval grey number, Which is posed by professor Deng Julong, We have already given it abstract definition in pape[3], i.e. the grey number as follow is called interval grey number:

$$\mu(x)=\begin{cases}\{1\}, x\in[a,b]\\ \{o\}, x\in[a,b],\end{cases} \qquad a,b\in R \text{ and } a<b.$$

We denote it as [a,b], and where a,b are said respectively left and right extreme point. It is easy to know that the interval grey number is one special fuzzy number.

For the further research of the grey equation, grey matrix, etc. , We are now trying generalize it as follows:

Definition 1.

(1) If $a<b$, for $a,b\in R$, interval $[a,b]$, $(a,b]$, $[a,b)$ and (a,b) are all called interval grey number, and then can be respectively denoted as $I[a,b]$, $I(a,b]$, $I[a,b)$ and $I(a,b)$.

(2) If $a>b$, with $a,b\in R$, interval $[a,+\infty)$, $(-\infty,b]$; $(a,+\infty)$ and $(-\infty,b)$; $[a,+\infty)$ and $(-\infty,b)$; $(a,+\infty)$ and $(-\infty,b]$ are all called interval grey number. We write them respectively as $[a,b]$, (a,b), $[a,b)$, $(a,b]$ or $I[a,b]$, $I(a,b)$, $I[a,b)$, $I(a,b]$.

In a word .For any $a,b\in R$, $I[a,b]$, $I(a,b)$, $I[a,b)$, $I(a,b]$ are all called interval grey number ,and which can de denoted as la,b. Where a,b are respectively called left extreme point and right extreme point.

Definition 2.

(1) If $a<b$, then the interval grey number la,b is called a finite interval grey number.

(2) if $a>b$, then the interval grey number la,b is called infinite interval grey number.

From the point of intuitive view, the interval grey number la,b is a remainder part of which lb,a is removed from the real line R, or an interval that surround the infinite point.

When a=b.then Ia,b=Ia,a=[a,a], i.e. grey number [a,a]. For $x \in R$, it can be all written as [x,x], i.e. Ix,x. When x=o, Io,o=[o,o].

Definition 3. Io,o is called null interval grey number.

Definition 4. If $a \neq o$ and $b \neq o$, the interval grey number Ia,b and I1/a,1/b are called reversal interval grey numbers to one another. Where each one is said to the reverse interval grey number of the other. The reverse interval grey number of Ia,b can be written as $\overline{Ia,b}$ i.e. $\overline{Ia,b}$=I1/a,1/b.

Definition 5. Interval grey numbers Ia,b,I-a,-b are called mutually opposite interval grey number, in which one is called another's opposite interval grey number. The opposite interval grey number of Ia,b can be denoted by -Ia,b, i.e. -Ia,b=I-a,-b.

For convenience , let $I(r) = \{[a,b] \mid a,b \in R, a < b\}$, $I(R) = \{Ia,b \mid a,b \in R\}$. Obviously , I(r) I(R), this is to say , if a<b , for any $a,b \in R$, the interval grey number [a,b] is a particular case of the Ia,b and I(R) is a generalization of the I(r).

II. Operation Of Interval Grey Number

on the basic of the generalization of interval grey number (or special fuzzy number), We are now trying to discuss their operations.

When x=o.we define $1/x=\infty$, $o/x=o$.

Definition 6. For any Ia,b, $Ic,d \in I(R)$.

(1) Addition ($\oplus$)

$$Ia,b \oplus Ic,d = \{x+y \mid x \in Ia,b, y \in Ic,d\} .$$

(2) Subtruction ($\ominus$)

$$Ia,b \ominus Ic,d = \{x-y \mid x \in Ia,b, y \in Ic,d\} .$$

(3) Multiplication ($\odot$)

$$Ia,b \odot Ic,d = \{x.y \mid x \in Ia,b, y \in Ic,d\} .$$

(4) Division ($\otimes$)

$$Ia,b \otimes Ic,d = \{x/y \mid x \in Ia,b, y \in Ic,d\} .$$

Where $c \neq o$, $d \neq o$.

Now let's reseach the laws of the operations.

(1) Addition

(I) The sum of two finite interval grey numbers is still a finite interval grey number, and

$$Ia,b \oplus Ic,d = Ia+c,b+d.$$

(Ⅱ) A finite interval grey number plus an infinite interval grey number give an infinite interval grey number,and

$$Ia,b \oplus Ic,d = Ia+c,b+d.$$

Where $a>b, c<d$. When $a+c<b+d$, Ia,b plus Ic,d gives is the whole real axis R.

(Ⅲ) The sum of two infinite interval grey number is the whloe axis R,i.e.

$$Ia,b \oplus Ic,d = R \quad \text{,where } a>b, c>d.$$

(2) Subtraction

(Ⅰ) The difference of two infinite interval grey numbers is the whole real axis.

(Ⅱ) The difference of two finite interval grey number is finite interval grey number, i.e.

$$Ia,b \ominus Ic,d = Ia-c,b-d.$$

Where $a<b$, $c<d$.

(Ⅲ) An infinite interval grey number subtract a finite interval grey number is an infinite interval grey number ,too.i.e.

$$Ia,b \ominus Ic,d = Ia-d,b-c \qquad a>b, c>d.$$

That is to say,the difference of any two interval grey numbers is equal to that subtracted interval grey number adds the opposite number of the subtracting interval grey number ,except for two infinite interval grey numbers.

(3) Multiplication

(Ⅰ) The product of any interval grey number time null interval grey number is a null interval grey number.

(Ⅱ) The product of any infinite interval grey number time a nonull finite interval grey number is an infinite interval grey number.

When the signs of left and right extreme points of a finite interval grey number are opposite,its product is the whole real axis.
When the signs of both left and right extreme points of finite interval grey number are negative,then

$$Ia,b \odot Ic,d = Ie,f, \quad a<b<o, c>d.$$

where $e=\min\{ad,bd\}$, $f=\max\{ac,bc\}$.

When both the left and right extremes of the finite interval grey number are positive, then

$$Ia,b \odot Ic,d = Ie,f, o<a<b, c>d$$

where $e=\min\{ac,bc\}$, $f=\max\{ad,bd\}$.

(Ⅲ) The product of two infinite interval grey numbers is an infinite interval grey number ,i.e.

$$la,b \odot lc,d = le,f, \; a>b, c>d$$

where e=min {ac,bd} ,f=max {ad,bc} .

(Ⅳ) The product of two finite interval grey number is a finite interval grey number .i.e

$$la,b \odot lc,d = le,f \qquad a<b, c<d$$

where e=min {ac,ad,bc,bd} ,f=max {ac,ad,bc,bd} .

(4) Division

on the light of definition 6(4).

$$la,b \oslash lc,d = \{x/y \mid x \in la,b, y \in lc,d\} .$$

$$la,b \odot \overline{lc,d} = la,b \odot l1/a,1/c = \{x.1/y \mid x \in la,b, y \in lc,d\} .$$

Where $c \neq o, d \neq o$

hence, $la,b \oslash lc,d = la,b \odot \overline{lc,d}$

That is to say, that an interval grey number divided by a nonnull interval grey number is equal to that this one times the reversal grey number of the other nonnull one.

From the operations and operation laws shown above, we know easily that arithmetic operation of I(r) is held in I(R), then arithmetic operations of I(R) are the generalization of operations of interval grey number [a,b],with a<b.

Ⅲ. Properties Of The Interval Grey Number.

Theorem 1. For any $la,b \in I(R)$

$$la,b \oplus lo,o = la,b.$$
$$la,b \odot lo,o = lo,o.$$

Theorem.2. For any $a \in R, lc,d \in I(R)$

(1). $a \odot lc,d = lac,ad.$ if $a>o$
(2). $a \odot lc,d = lad,ac.$ if $a<o$

Theorem 3. If $lc,d \in I(R)$,for any $a \in R$,then

$$a \oplus lc,d = la+c,a+d.$$

Theorem 4. Addition and multiplication of the interval grey number are all satisfies commutative law, i.e.

$$la,b \oplus lc,d = lc,d \oplus la,b,$$
$$la,b \odot lc,d = lc,d \odot la,b$$

Theorem 5

(1) The addition operation of interval grey number satisfies association law, i.e.

$$la,b\oplus(lc,d\oplus le,f)=(la,b\oplus lc,d)\oplus le,f$$

(2) The multiplication operation of interval grey number satisfies association law, too, i.e.

$$la,b\odot(lc,d\odot le,f)=(la,b\odot lc,d)\odot le,f$$

Proof :

We prove (1) here.

(I) If la,b, lc,d, le,f are all finite interval grey number. by the operation rules of the addition.

$$\begin{aligned}&la,b\oplus(lc,d\oplus le,f)=la,b\oplus lc+e,d+f\\&=la+c+e,b+d+f=la+c,b+d\oplus le,f\\&=(la,b\oplus lc,d)\oplus le,f.\end{aligned}$$

Hence (1) hold.

(II) With la,b, lc,d, le,f, only one of them is a finite interval grey number. We may assume, Without loss of generality , that la,b is a finite interval grey number.

$$\begin{aligned}&la,b\oplus(lc,d\oplus le,f)=la,b\oplus(-\infty,+\infty)=\text{the whole real axis;}\\&(la,b\oplus lc,d)\oplus le,f=la+c,b+d\oplus le,f=\text{the whole real axis.}\end{aligned}$$

Hence ,(1) is true.

(III) With la,b, lc,d, le,f, Two of them are finite interval grey numbers. Without loss of generality, to suppose that la,b and lc,d are finite interval grey numbers.

$$\begin{aligned}&la,b\oplus(lc,d\oplus le,f)=la,b\oplus lc+e,d+f\\&=la+c+e,b+d+f=la+c,b+d\oplus le,f\\&=(la,b\oplus lc,d)\oplus le,f\end{aligned}$$

Hence, $la,b\oplus(lc,d\oplus le,f)=(la,b\oplus lc,d)\oplus le,f$.

(IV) Suppose that la,b, lc,d, le,f are all infinite interval grey numbers, It is easy to know,

$$\begin{aligned}la,b\oplus(lc,d\oplus le,f)&=\text{the whole real axis}\\&=(la,b\oplus lc,d)\oplus le,f\end{aligned}$$

this (1) is true.

Hence, the result (1) follows from mentioned above.

Similarly, We can prove (2).

Reference

[1] Deng Ju-Long, Fundamental nanners of grey system, HuZhong university of science and technology press 1985.

[2] Wu He-Qin Wang Qiny-Yin, The theory of Compound fuzzy sets, proceeding of NAFIPS'88.

[3] Wang Qiny-Yin Wu He-Qin , The concept of grey number and its property, proceeding of NAFIPS'88.