ON FUZZY SET OPERATIONS

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"The main questions of fuzzy set theory and fuzzy logic, arisen from its origin,is connected with conjuction, disjunction, negation and implication definitions. It became clear after important work by Dr.Zimmermann and his Aachen's colleagues that everuthing we need for many practical tasks solving is a parametric definition family which in case of need would assume some non-standard choice of operators, reflecting the characteristic features of specific supplement. The advantage of such approash is that avoiding fixed, specifically-independent definitions fuzzy set theory and fuzzy logic are reaching pluralism, increasing their flexibility and expressive possibilities" (L.A.Zadeh).

It's known that introduktion of triangular norms $\top$ and triangular conorms $\perp$ presented one of the feasible ways of solving this task [1] - [3].

Let's formulate the results.

1. Let two-place real function $H: I \times I \to I$, $I=[a,b]$, $0 \leq a < b < \infty$ satisfies the folloving condition: a)associativity, b)commutativity, c)non-decreasing in each argument, d) $H(0,z)=z$
Function $g:[0,1] \to [0,b]$ (function $f:[0,1] \to [0,b]$)single-placed continuosly strictly increasing(strictly decreasing) $g(0)=f(1)=0$.
Theorem 1. Two-placed real function

$$\perp : [0,1] \times [0,1] \to [0,1], \ \perp(x,y) = g^{-1}(min(g(1), \ H(g(x), g(y))))$$

is a triangular conorm.

Theorem 2. Two-place real function

$$T:[0,1]\times[0,1]\to[0,1]\,,\ \ T(x,y)=f^{-1}(\min(f(0),\ H(f(x),f(y))))$$

is a triangular norm.

Example 1.

$$T(x,y)=f^{-1}(\min(f(0),\ f(x)+f(y)(1+\lambda f(x)/f(0))))$$

$$\perp(x,y)=g^{-1}(\min(g(1),\ g(x)+g(y)(1+\lambda g(x)/g(1))))$$

here and further $-1<\lambda<\infty$.

Example 2.

$$T(x,y)=\max(0,\ (1+\lambda)(x^p+y^p-1)-\lambda x^p y^p)^{\frac{1}{p}}$$

$$T(x,y)=\max(0,\ \frac{1}{1+\lambda}(x^p+y^p-1)+\frac{\lambda}{1+\lambda}x^p y^p)^{\frac{1}{p}}$$

dual $\perp(x,y)=\min(1,\ x^p+y^p+\lambda x^p y^p)^{\frac{1}{p}}$

correspondingly relatively strong negation

$$n(x)=(1-x^p)^{\frac{1}{p}},\quad n(x)=((1-x^p)/(1+\lambda x^p)^{\frac{1}{p}}$$

here and further $p>0$.

2. Let L is set, $L\neq\emptyset$. $\mathcal{B}$ is Borel algebra of L . Denote μ fuzzy measure, presented by Sugeno [5]. As well known [2], [6] fuzzy measure conjunction two set presentation as

$$\mu(A\cup B)=\perp(\mu(A),\mu(B)),\quad A\cap B=\emptyset,\quad \forall A,B\in\mathcal{B}.$$

We construct now fuzzy measure generalise Sugeno's measure

$$R_\lambda(A\cup B)=g^{-1}(\min(g(1),\ g\circ\mu(A)+g\circ\mu(B)(1+\lambda g\circ\mu(A)/g(1)))).$$

Then

$$R_\lambda(\bar{A}) = g^{-1}((g(1) - g\circ\mu(A))/(1+\lambda g\circ\mu(A)/g(1))), \quad g(1) < \infty .$$

If $\lambda = 0$ we hawe Weber measure [6] .

3. Let F is function meaning with property of theoretic-probability of distribution function. Using such function, for $\forall (a,b) \subset R$ (R is real line), generalise Sugeno distribution F_S as folloving

$$R_\lambda((a,b)) = g^{-1}((g\circ F(b) - g\circ F(a))/(1+\lambda g\circ F(a)/g(1))), \quad g(1) < \infty .$$

When $F(x) = x$ put

$$\mu_\lambda((a,b)) = g^{-1}((g(b) - g(a))/(1+\lambda g(a)/g(1)))$$

Denote through $E_\mu(h)$ Sugeno integral [5] of measurable function $h : L \to [0,1]$ with respect to a fuzzy measure μ .

Theorem 3. If strong negation function

$$n(x) = g^{-1}((g(1) - g(x))/(1+\lambda g(x)/g(1)))$$

then $\quad n(E_{R_\lambda}(h)) = E_{R_\lambda}(n(h))$.

Theorem 4.

$$E_{\mu_\lambda}(h) = g^{-1}((\sqrt{1+\lambda} - 1)\, g(1)/\lambda)$$

$$E_{\mu_0}(h) = g^{-1}(g(1)/2) .$$

Example 3. If $g(x) = x^p$ then $E_{\mu_0}(h) = (\frac{1}{2})^{\frac{1}{p}}$

If $g(x)=1-(1-x)^{p}$ then $E_{\mu_0}(h)=1-(\frac{1}{2})^{\frac{1}{p}}$.

4. Let two-place real function $N: I\times I\to I$ satisfies conditions: b), c), e)contininious, f) $N(0,0)=0$, $N(b,b)=b$, $g(1)=b$,

g) $N(r,z)\in[min(r,z), max(r,z)]$, $N \overline{\in} \{min, max\}$

and M is averaging operator [7]. Then two-place function

$$Mg:[0,1]\times[0,1]\to[0,1], \quad Mg(x,y)=g^{-1}(N(g(x), g(y)))$$

is averaging operator. If n is strong negation function, then

$$M_n(x,y)=n^{-1}\circ g^{-1}(N(g\circ n(x), g\circ n(y)))$$

also is averaging operator. Let $K: I\times I\to I$ satisfies conditions: b)-e) and when n is strong negation function, then

$$g^{-1}(K(g(x), g(y)))=n^{-1}\circ g^{-1}(K(g\circ n(x), g\circ n(y))).$$

K is named self-dual comparatively strong negation n.

Then

$$M_g(x,y)=g^{-1}(K(g(x), g(y))),$$

$$M_n(x,y)=n^{-1}\circ g^{-1}(K(g\circ n(x), g\circ n(y)))$$

is equal: $M=M_g=M_n$. In this case is named their averaging operator self-dual comparatively strong negation n.

Example 4. If $\beta+\beta_1=1$, $\beta>0$ then

$$M(x,y)=(\beta x^{p}+\beta_1 y^{p})^{\frac{1}{p}}, \quad n(x)=(1-x^{p})^{\frac{1}{p}}$$

$$M(x,y)=1-(\beta(1-x)^{p}+\beta_1(1-y)^{p})^{\frac{1}{p}}, \quad n(x)=1-(1-(1-x)^{p})^{\frac{1}{p}}$$

$$M(x,y)=\frac{(1+\lambda x)^{\beta}(1+\lambda y)^{\beta_1}-1}{\lambda}, \quad n(x)=\frac{1-x}{1+\lambda x}.$$

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