Epistemic entrenchment, partial inconsistency and abnormality in possibilistic logic *

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1 - Introduction

All the pieces of information stored in a knowledge base are not always regarded as equally certain. Quite often a part of the information is not completely certain since it is based on incomplete evidence or it corresponds to rules liable to have exceptions. Possibilistic logic (see Dubois and Prade, 1988), through the use of so-called necessity measures, offers a way of grading certainty on a numerical scale. As recalled in the next section, these necessity measures are the unique numerical counterpart of qualitative ordering relations which model "is at least as certain as". These qualitative foundations of necessity measures are in agreement with the fact that when assessing degrees of certainty, only the ordering of these numbers is sometimes really meaningful. In section 3 we point out that the axioms satisfied by an ordering relation underlying a qualitative necessity measure are equivalent to those recently proposed by Gärdenfors and Mackinson (1988) for modelling epistemic entrenchment in order to first question the less entrenched belief in a revision process. Then it is not too much surprizing that the capacity of possibilistic logic to handle non-monotonic reasoning resides in its ability to inhibit the least entrenched pieces of information in partially inconsistent knowledge bases, as explained in section 4. Finally, section 5 shows that possibilistic reasoning is also in agreement with the idea of minimizing abnormality in commonsense reasoning.

N.B.: Proofs, details and examples are omitted in this extended abstract for the sake of brevity.

2 - Necessity measures and their qualitative counterpart

Let $\mathfrak{B}$ be a (finite) Boolean algebra of propositions (denoted by **a, b, c, ...**), equipped with an ordering relation $\geq$; $\mathbf{a} \geq \mathbf{b}$ means "**a** is at least as certain as **b**". By definition the relation $\geq$ satisfies the following requirements (Dubois, 1986, 1988), for any **a**, **b** and **c**

A1	• reflexivity	$\mathbf{a} \geq \mathbf{a}$
A2	• completeness	$\mathbf{a} \geq \mathbf{b}$ or $\mathbf{b} \geq \mathbf{a}$
A3	• transitivity	if $\mathbf{a} \geq \mathbf{b}$ and $\mathbf{b} \geq \mathbf{c}$ then $\mathbf{a} \geq \mathbf{c}$
A4	• non-triviality	$\mathbb{1} > \mathbb{O}$, where $\mathbb{1}$ (resp. : $\mathbb{O}$) is the greatest (resp. : least) element in $\mathfrak{B}$, and $\mathbf{a} > \mathbf{b}$ means "$\mathbf{a} \geq \mathbf{b}$ and not ($\mathbf{b} \geq \mathbf{a}$)"
A5	• certainty of tautology	$\mathbb{1} \geq \mathbf{a}$
A6	• stability under conjunction	$\forall \mathbf{a}$, if $\mathbf{b} \geq \mathbf{c}$ then $\mathbf{a} \wedge \mathbf{b} \geq \mathbf{a} \wedge \mathbf{c}$

Such a relation is called a "qualitative necessity measure". The associated qualitative possibility measure is defined by $\succeq$ with $\mathbf{a} \succeq \mathbf{b}$ if and only if $\neg\mathbf{b} \geq \neg\mathbf{a}$ and reads "**a** is at least as possible as **b**".

A (numerical) necessity measure (see Dubois and Prade, 1988) is a mapping N from $\mathfrak{B}$ to the real interval [0,1] such that

$$N(\mathbb{0}) = 0,\ N(\mathbb{1}) = 1$$

$$\text{and } \forall \mathbf{a}, \forall \mathbf{b},\ N(\mathbf{a} \wedge \mathbf{b}) = \min(N(\mathbf{a}), N(\mathbf{b}))$$

A straightforward consequence is that $\forall \mathbf{a}$, $\min(N(\mathbf{a}), N(\neg\mathbf{a})) = 0$. A dual possibility measure Π is defined from N by $\forall \mathbf{a}$, $\Pi(\mathbf{a}) = 1 - N(\neg\mathbf{a})$. Clearly Π is such that $\Pi(\mathbb{0}) = 0$, $\Pi(\mathbb{1}) = 1$ and $\forall \mathbf{a}$, $\forall \mathbf{b}$, $\Pi(\mathbf{a} \vee \mathbf{b}) = \max(\Pi(\mathbf{a}), \Pi(\mathbf{b}))$, i.e. Π is a possibility measure in the sense of Zadeh (1978).

N.B. : We do not have $\forall \mathbf{a}, \forall \mathbf{b}$, $N(\mathbf{a} \vee \mathbf{b}) = \max(N(\mathbf{a}), N(\mathbf{b}))$ but only $N(\mathbf{a} \vee \mathbf{b}) \geq \max(N(\mathbf{a}), N(\mathbf{b}))$. Forcing the equality for all **a** and **b** entails that N is just a standard truth assignment function, i.e. such that $\forall \mathbf{a}$, $N(\mathbf{a}) = 1$ or $N(\mathbf{a}) = 0$.

A function g from $\mathfrak{B}$ to [0,1] is compatible with an ordering relation $\geq$ on $\mathfrak{B}$ if and only if $\mathbf{a} \geq \mathbf{b} \Leftrightarrow g(\mathbf{a}) \geq g(\mathbf{b})$.

Then the following results hold in finite settings :

- A qualitative necessity measure is compatible with g if and only if g is a necessity measure. Particularly, A1-A6 imply $\mathbf{a} \sim (\mathbf{a} \wedge \mathbf{b})$ as soon as $\mathbf{b} \geq \mathbf{a}$, where $\mathbf{a} \sim \mathbf{b}$ means "$\mathbf{a} \geq \mathbf{b}$ and $\mathbf{b} \geq \mathbf{a}$".

- Let $\vdash$ be the partial ordering of $\mathfrak{B}$ (induced by the Boolean structure) then $\mathbf{b} \vdash \mathbf{a} \Rightarrow \mathbf{a} \geq \mathbf{b}$ and $\mathbf{a} \succeq \mathbf{b}$.

- If $\geq$ is compatible with N then $\succeq$ is compatible with $\Pi(.) = 1 - N(\neg .)$, i.e. the dual possibility measure.

3 - Epistemic entrenchment and qualitative necessity measures

Epistemic entrenchment is a way of assigning priorities to sentences in a knowledge base in order to facilitate revision and contraction of the base. It is modelled by an ordering relation between the sentences : **a** ≤ **b** means "**b** is at least epistemically entrenched as **a**". The relationship between the epistemic entrenchment relation (Gärdenfors and Mackinson, 1988) and necessity measures is now clear. Indeed transitivity is in both notions ; maximality (if **b** ≤ **a** for all **b**, then ⊢**a**) is A5, dominance (if **a** ⊢ **b** then **a** ≤ **b**) is a consequence of A1-A6 ; conversely completeness (A2) is a consequence of epistemic entrenchment axioms. Dominance and conjunctiveness (for any **a** and **b**, **a** ≤ **a** ∧ **b** or **b** ≤ **a** ∧ **b**) imply **a** ~ **a** ∧ **b** (if **b** ≥ **a**) which is equivalent to A6 if A5 is accepted as well. Although starting with different axioms and different motivations, the same kind of relation is obtained.

4 - Dealing with uncertain knowledge bases

In Dubois and Prade (1987), the resolution principle has been extended into

$$\frac{N(\mathbf{a} \vee \mathbf{b}) \geq \alpha \qquad N(\neg\mathbf{a} \vee \mathbf{c}) \geq \beta}{N(\mathbf{b} \vee \mathbf{c}) \geq \min(\alpha,\beta)}$$

and the particularization rule into

$$\frac{N(\forall x, p(x)) \geq \alpha}{N(p(a)) \geq \alpha}$$

An uncertain knowledge base is a set $\mathcal{K} = \{(C_i\ \alpha_i) \mid i = 1,n,\ \alpha > 0\}$ where α_i is a lower bound on $N(C_i)$ where C_i denotes a clause. $\mathcal{K}$ is consistent with respect to necessity measures if and only if the set of ordinary clauses $\{C_i \mid (C_i\ \alpha_i) \in \mathcal{K}\}$ is consistent in the usual sense ; this is equivalent to not being able to deduce $\mathbb{O}$ from $\mathcal{K}$ using the extended resolution, such that $N(\mathbb{O}) > 0$. Then using a refutation method (Dubois and Prade, 1987) we can produce the best lower bound of the necessity of any proposition **p** to evaluate with respect to $\mathcal{K}$ (the refutation method consists in adding $(\neg \mathbf{p}\ \ 1)$ to $\mathcal{K}$ and any weight attached to a derived empty clause is a lower bound of the necessity of the refuted proposition). Then it can be proved that

$$\text{if } \mathcal{K} \vdash N(\mathbf{p}) \geq \alpha$$

$$\text{then } \mathcal{K} \cup \{(C_{n+1}\ \ \alpha_{n+1})\} \vdash N(\mathbf{p}) \geq \beta$$

with $\beta \geq \alpha$, provided that $\mathcal{K} \cup \{(C_{n+1}\ \ \alpha_{n+1})\}$ remains consistent. This a monotonic behaviour.

A semantic has been defined for clauses weighted by lower bounds of a necessity measure (Dubois, Lang and Prade, 1988). If **p** is a closed formula, $M(\mathbf{p})$ the set of the models of **p**, then the models of $(\mathbf{p}\ \ \alpha)$ will be defined by a fuzzy set $M(\mathbf{p}\ \ \alpha)$ with a membership function

$$\mu_{M(\mathbf{p}\ \alpha)}(I) = 1 \text{ if } I \in M(\mathbf{p})$$

$$= 1 - \alpha \text{ if } I \in M(\neg \mathbf{p}).$$

Then the fuzzy set of the models of a knowledge base $\mathcal{K} = \{C^*_1, C^*_2, \ldots, C^*_n\}$, where C^*_i is a closed formula with its weight, will be the intersection of the fuzzy sets $M(C^*_i)$, i.e. $\mu_{M(\mathcal{K})}(I) = \min_{i=1,\ldots,n} \mu_{M(C^*_i)}(I)$. The <u>consistency</u> degree of $\mathcal{K}$ will be defined by $c(\mathcal{K}) = \max_I \mu_{M(\mathcal{K})}(I)$; it estimates the degree to which the set of models of $\mathcal{K}$ is not empty. The quantity $\mathrm{Inc}(\mathcal{K}) = 1 - c(\mathcal{K})$ will be called degree of <u>inconsistency</u> of $\mathcal{K}$.

By contrast, when $\mathcal{K}$ is not fully consistent but only consistent to the degree $1 - \alpha < 1$, i.e.

$\mathcal{K}$ is α-inconsistent, there is room for non-monotonicity (see Dubois, Lang and Prade, 1988). When $\mathcal{K}$ is α-inconsistent, i.e. $N(\bot) \geq \alpha > 0$, it is still possible to infer non-trivial conclusions from $\mathcal{K}$, namely all consequences **p** of $\mathcal{K}$ such that $N(\mathbf{p}) \geq \beta > \alpha$, using resolution and refutation on $\mathcal{K} \cup \{(\neg \mathbf{p}\ \ 1)\}$. Indeed, if $\beta > \alpha$, there exists a consistent sub-base $\mathcal{S}$ of $\mathcal{K}$ from which we can infer $(\mathbf{p}\ \ \beta)$ by resolution, and then we will consider the proof of $(\mathbf{p}\ \ \beta)$ as valid since, this refutation uses only clauses whose necessity degree is greater or equal to β, i.e. strictly greater than α. Everything occurs as if all pieces of information $(\mathbf{b}\ \ \gamma)$ with $\gamma \leq \alpha$ were occulted, and the proof paths only consider $\{(\mathbf{b}\ \ \beta) \in \mathcal{K} \mid \beta > \alpha\}$ as the actual knowledge base (it is a consistent one !). In the terminology of entrenchment, the least entrenched pieces of information are inhibited, only a maximal consistent sub-base of strongly entrenched pieces of information remain. The relationship between the non-monotonic behavior of partially inconsistent possibilistic knowledge bases and revision processes as studied by Gärdenfors and Mackinson (1988), will be investigated.

5 - Minimizing abnormality

A clause like $(\neg p(x) \vee q(x)\ \ \alpha)$, once instanciated on a particular x, say a, means $N(\neg p(a) \vee q(a)) \geq \alpha$. In other words, it expresses that there is a possibility at most equal to $1 - \alpha$ (i.e. $\Pi(p(a) \wedge \neg q(a)) \leq 1 - \alpha$) that a particular x is an exception of the rule "if x satisfies p, then it satisfies q". Another way of handling a rule with (potential) exceptions is to introduce an abnormality predicate, say "ab", specific of the rule, and to state the totally <u>certain</u> rule

$$(\neg p(x) \vee q(x) \vee ab(x)\ \ 1)$$

i.e. "if x satisfies p, it satisfies q or it is abnormal", and to add to the knowledge base the default assumption

$$(\neg ab(x)\ \ \alpha)$$

i.e. $\forall a$, $N(\neg ab(a)) \geq \alpha$ (we are at least certain at degree α that a given x is not a priori abnormal). Then the extended resolution principle recalled in section 3 enables us, from the weighted clauses

$$(\neg p(x) \vee q(x) \vee ab_1(x) \ \ 1)$$

$$(p(x) \vee r(x) \vee ab_2(x) \ \ 1)$$

$$(\neg ab_1(x) \ \ \alpha)$$

$$(\neg ab_2(x) \ \ \beta)$$

to derive the weighted clause

$$(q(x) \vee r(x) \ \ \min(\alpha,\beta))$$

Thus we obtain the same results that the one we can get starting with the weighted clauses $(\neg p(x) \vee q(x) \ \ \alpha)$ and $(p(x) \vee r(x) \ \ \beta)$. Then the search for the largest weight attached to a derived empty clause in a refutation procedure will correspond to try to obtain the empty clause using only the most certain clauses of the form $(\neg ab_i(x) \ \ \alpha_i)$, i.e. maximizing the normality or equivalently minimizing the abnormality.

However an advantage of this approach is that we can express relations between abnormality predicates. We may for instance have the following piece of information

$$(\neg ab_1(x) \vee ab_2(x) \ \ \gamma)$$

which means that if x is an exception to rule 1, it is generally an exception to rule 2. Then we can minimize a <u>contextual</u> abnormality (provided that the corresponding knowledge is available) as advocated by Pearl (1987) in the case of conditional probabilities, rather than simply minimizing all abnormal events.

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