FUZZY PROBABILITY IMPORTANCE IN FAULT TREE ANALYSIS

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ABSTRACT

This paper develops the probability importance in general reliability into the fuzzy probability importance in fuzzy reliability by means of basic concepts and principles of fuzzy mathematics. In this paper author derives the expression of the fuzzy probability importance.

KEYWORD

Fuzzy probability importance.

INTRODUCTION

The importance is an important concept in reliability engineering. It plays a directive role in reliability prediction, reliability assignation, operation of an equipment and maintenance of an equipment.

At present an importance is classified as about nine kinds as a matter of record at home and abroad. This paper only has a discussion on developing general probability importance into fuzzy probability importance.

This paper only considers the indexes of FA mode.

1. DEFINITION OF FUZZY PROBABILITY IMPORTANCE

An effect of different failure mode in fault tree on successful operation of a system is different. An importance is a measure, that expresses effect of failure mode on successful operation of system. General probability importance (I_K) is a measure, that expresses a decrease of system unreliability from failure state of kth element to successful operation state of the element, that is,

$$I_k = F_S(F_k=1) - F_S(F_k=0) \quad (1)$$

where F_k is unreliability of kth element.

$F_S(F_k=1)$ is unreliability of the system when kth element has occurred failure.

$F_S(F_k=0)$ is unreliability of the system when kth element is successful operation.

The definition of fuzzy probability importance is expressed as follow: "Fuzzy probability importance $(\underset{\sim}{I}_k)$ is a measure, that expresses a decrease of system fuzzy unreliability relating to some fuzzy performance subset $(\underset{\sim}{A}_i)$ from some fuzzy failure state $(\underset{\sim}{B}_j)$ of kth element to the fuzzy performance state $(\underset{\sim}{A}_i)$ of kth element", that is,

$$\underset{\sim}{I}_k = \underset{\sim}{F}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{B}_j) - \underset{\sim}{F}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{A}_i) \quad (2)$$

where $\underset{\sim}{F}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{B}_j)$ is system fuzzy unreliability relating to fuzzy performance subset $\underset{\sim}{A}_i$ when kth element has occurred fuzzy failure $\underset{\sim}{B}_j$,

$\underset{\sim}{F}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{A}_i)$ is system fuzzy unreliability relating to fuzzy performance subset $\underset{\sim}{A}_i$ when kth element is fuzzy performance state $\underset{\sim}{A}_i$.

2. EXPRESSION OF FUZZY PROBABILITY IMPORTANCE

Two fuzzy unreliability of a system in Eq.(2) are expressed as follow in terms of fuzzy reliability

$$\underset{\sim}{F}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{B}_j) = 1 - \underset{\sim}{R}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{B}_j) \quad (3)$$

$$\underset{\sim}{F}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{A}_i)=1-\underset{\sim}{R}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{A}_i) \tag{4}$$

where $\underset{\sim}{R}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{B}_j)$ is that fuzzy reliability of a system which is opposite from $\underset{\sim}{F}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{B}_j)$,

$\underset{\sim}{R}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{A}_i)$ is that fuzzy reliability of a system which is opposite from $\underset{\sim}{F}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{A}_i)$.

Substituting Eqs. (3) and (4) into Eq.(2), we obtain

$$\underset{\sim}{I}_k=\underset{\sim}{R}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{A}_i)-\underset{\sim}{R}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{B}_j) \tag{5}$$

Therefore fuzzy probability importance is alsomexprwssed as an increase of fuzzy reliability of a system.

According to the definition of fuzzy conditional probability[2] we have

$$\underset{\sim}{R}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{B}_j)=P(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{B}_j) = \frac{P[(\text{"S"}=\underset{\sim}{A}_i)\wedge(\text{"k"}=\underset{\sim}{B}_j)]}{P(\text{"k"}=\underset{\sim}{B}_j)} \tag{6}$$

$$\underset{\sim}{R}_S(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{A}_i)=P(\text{"S"}=\underset{\sim}{A}_i \mid \text{"k"}=\underset{\sim}{A}_i) = \frac{P[(\text{"S"}=\underset{\sim}{A}_i)\wedge(\text{"k"}=\underset{\sim}{A}_i)]}{P(\text{"k"}=\underset{\sim}{A}_i)} \tag{7}$$

where the sign $\wedge$ denotes algebraic product.

By means of fuzzy reliability theory[1] we have

$$P[(\text{"S"}=\underset{\sim}{A}_i)\wedge(\text{"k"}=\underset{\sim}{B}_j)]=\mu_{\underset{\sim}{A}_i}[R_S(\text{"k"}=\underset{\sim}{B}_j)]R_S(\text{"k"}=\underset{\sim}{B}_j) \tag{8}$$

$$P[(\text{"S"}=\underset{\sim}{A}_i)\wedge(\text{"k"}=\underset{\sim}{A}_i)]=\mu_{\underset{\sim}{A}_i}[R_S(\text{"k"}=\underset{\sim}{A}_i)]R_S(\text{"k"}=\underset{\sim}{A}_i) \tag{9}$$

$$\begin{aligned} P(\text{"k"}=\underset{\sim}{B}_j) &= \underset{\sim}{F}_k(\text{"k"}=\underset{\sim}{B}_j) \\ &= 1-\underset{\sim}{R}_k(\text{"k"}=\underset{\sim}{B}_j) \\ &= 1-\mu_{\underset{\sim}{B}_j}(R_k)R_k \end{aligned} \tag{10}$$

$$P(\text{"k"}=\underset{\sim}{A_i})=\underset{\sim}{R_k}(\text{"k"}=\underset{\sim}{A_i})$$

$$=\mu_{\underset{\sim}{A_i}}(R_k)R_k \tag{11}$$

where R_k is general reliability of kth element,

$\mu_{\underset{\sim}{B_j}}(R_k)$ is degree of membership of R_k in $\underset{\sim}{B_j}$,

$\mu_{\underset{\sim}{A_i}}(R_k)$ is degree of membership of R_k in $\underset{\sim}{A_i}$,

$\mu_{\underset{\sim}{A_i}}\left[R_S(\text{"k"}=\underset{\sim}{B_j})\right]$ is degree of membership of $R_S(\text{"k"}=\underset{\sim}{B_j})$ in $\underset{\sim}{A_i}$,

$\mu_{\underset{\sim}{A_i}}\left[R_S(\text{"k"}=\underset{\sim}{A_i})\right]$ is degree of membership of $R_S(\text{"k"}=\underset{\sim}{A_i})$ in $\underset{\sim}{A_i}$,

$R_S(\text{"k"}=\underset{\sim}{B_j})$ is general reliability of system when kth element has occurred fuzzy failure $\underset{\sim}{B_j}$,

$R_S(\text{"k"}=\underset{\sim}{A_i})$ is general reliability of system when kth element is fuzzy performance state $\underset{\sim}{A_i}$.

Evidently $R_S(\text{"k"}=\underset{\sim}{B_j})$ and $R_S(\text{"k"}=\underset{\sim}{A_i})$ are accurate probability of clear events, they are solved by means of general reliability theory[3]. Every element or system only has two state ("failure" and "operation") in general reliability theory. Therefore $R_S(\text{"k"}=\underset{\sim}{B_j})$ is general reliability of system when kth element has occurred failure, and $R_S(\text{"k"}=\underset{\sim}{A_i})$ is general reliability of system when kth is successful operation, namely

$$R_S(\text{"k"}=\underset{\sim}{B_j})=R_S(\text{"k"}=1) \tag{12}$$

$$R_S(\text{"k"}=\underset{\sim}{A_i})=R_S(\text{"k"}=0) \tag{13}$$

Finally substitution of Eqs. (6) - (13) into Eq.(5) yields

$$\underset{\sim}{I}_k=\frac{\mu_{\underset{\sim}{A}_i}[R_S(''k''=0)]R_S(''k''=0)}{\mu_{\underset{\sim}{A}_i}(R_k)R_k}-\frac{\mu_{\underset{\sim}{A}_i}[R_S(''k''=1)]R_S(''k''=1)}{1-\mu_{\underset{\sim}{B}_j}(R_k)R_k} \qquad (14)$$

this equation is general expression of fuzzy probability importance.

3. EXAMPLE

The Fault Tree and Reliability Block Diagram of Some System are Shown in Fogure. Assuming that

$\underset{\sim}{A}_i$ denotes the "very reliable",

$\underset{\sim}{B}_j$ denotes the "more large failure",

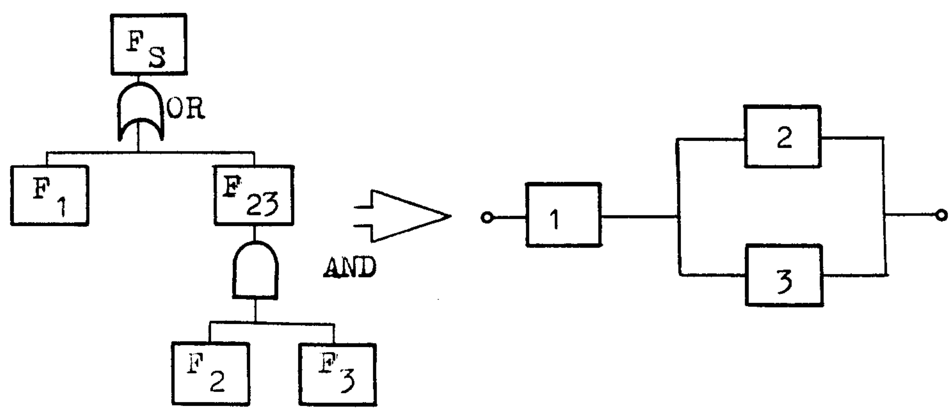
$R_1=0.998$, $R_2=R_3=0.946$,

$\mu_{\underset{\sim}{A}_i}(0.998)=0.90$, $\mu_{\underset{\sim}{A}_i}(0.997)=0.88$,

$\mu_{\underset{\sim}{A}_i}(0.946)=0.70$, $\mu_{\underset{\sim}{A}_i}(0.944)=0.65$,

$\mu_{\underset{\sim}{B}_j}(0.944)=0.40$,

determine the fuzzy probability importance of kth element (k=1,2,3).



Fualt Tree Reliability Block Diagram

Figure

The system surely occurs failure when 1st element occurs failure (i.e. "1"=1), since 1st element is a series, that is,

$$R_S("1"=1)=0$$

Reliability of the system depend on reliability of parallel subsystem consisted of 2nd element and 3rd element when 1st element is successful operation (i.e. "1"=0), that is,

$$\begin{aligned} R_S("1"=0) &= 1-(1-R_2)(1-R_3) \\ &= 1-(1-0.946)^2 = 0.997 \end{aligned}$$

According as Eq.(14) fuzzy probability importance of 1st element is

$$\underset{\sim}{I}_1 = \frac{0.88 \times 0.997}{0.9 \times 0.998} = 0.98$$

Reliability of the system depend on reliability of series group consisted of 1st element and 3rd element when 2nd element occurs failure (i.e. "2"=1), that is,

$$R_S("2"=1)=R_1 \times R_3=0.998 \times 0.946=0.944$$

Reliability of the system only depend on reliability of 1st element when 2nd element is successful operation (i.e. "2"=0), that is,

$$R_S("2"=0)=R_1=0.998$$

According as Eq.(14) fuzzy probability importance of 2nd element is

$$\underset{\sim}{I}_2 = \frac{0.9 \times 0.998}{0.7 \times 0.946} - \frac{0.65 \times 0.944}{1-0.4 \times 0.946} = 0.37$$

The same as 2nd element, fuzzy probability importance of 3rd element is

$$\underset{\sim}{I}_3 = \underset{\sim}{I}_2 = 0.37$$

1st element is more important than 2nd element or 3rd elemrnt, since 1st element is a series, and a group consisted of 2nd element and 3rd element is a parallel. Then

$$\underset{\sim}{I}_1 > \underset{\sim}{I}_2 \text{ or } \underset{\sim}{I}_3$$

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