ON INDUCED FUZZY TOPOLOGICAL GROUPS

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In this paper the separability,completeness,subgroups and quotient groups of induced fuzzy topological groups are disscussed.

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1. On the spaces of induced fuzzy topological groups

Let A be a fuzzy set in X with membership function $\mu_A(x)$. For any $r\in[0,1)$, $A_r=\{x \mid \mu_A(x)>r\}$ denotes the strong r-cut and r* denotes the fuzzy set in X with membership function $\mu_{r*}(x)=r$ for all $x\in X$.

Proposition 1.1. Let (X,T) be an ordinary topological group and $F(T)=\{A \mid$ A-a fuzzy set in X and for all $r\in[0,1)$, $A_r\in T\}$ .Then (X,F(T)) is a fuzzy topological group.We call it induced fuzzy topological group on (X,T).

Proof. It is clear that (X,F(T)) is a fuzzy topological space. Hence the only thing which remains to be,proved is that the operations which X possessed are fuzzy continuous relative to the fuzzy topology F(T).

(i) For all $a,b\in X$ and any open Q-neighboohood W' of

$(ab)_\lambda$, since $\mu_{W'}(ab) > 1-\lambda$ we can choose a $r \in (0,1]$ such that $1-\lambda < r < \mu_{W'}(ab)$. Then $W = W'_r$ is an open neighborhood of (ab) in (X,T). Hence in (X,T) there are open neighborhoods U of a and V of b such that $UV \subset W$.

Putting $U' = U \cap r^*$ and $V' = V \cap r^*$. Now in $(X,F(T))$, U' and V' are Q-neighborhoods of a_λ and b_λ respectively and it is easy to check that $U'V' \subset W'$.

(ii) For any $a \in X$ and any open Q-neighborhood V' of a^{-1}_λ quite similarly we can find a Q-neighborhood U' of a_λ such that $U'^{-1} \subset V'$, this completed the proof.

Definition 1.1. Let (X,J) be a fuzzy topological group. We say it belongs to type (QL) iff there exist a family of fuzzy sets $\mathcal{U} = \{U\}$ in X such that for any $\lambda \in (0,1]$ and any decreasing sequence $\{r_i\}$, where $1-\lambda < r_i \leq 1$ and $r_i \to 1-\lambda$, $\mathcal{U}_\lambda = \{U \cap r_i^* \mid U \in \mathcal{U}\}$ is a Q-neighborhood base of e_λ. We call the family $\mathcal{U}$ a model of base of the fuzzy topological group (X,J).

Proposition 1.2. Any induced fuzzy topological group $(X,F(T))$ on an ordinary topological group (X,T) belongs to type (QL).

Propositon 1.3. If a fuzzy topological group (X,J) belongs to type (QL) and $\mathcal{U} = \{U\}$ is a neighborhood base of e . Then $\mathcal{U}$ is a model of base of (X,J).

Proof. It is similar to the proof of Theorem 1.2 in [5] .

Proposition 1.4. Let $(X,F(T))$ be a induced fuzzy topological group on an ordinary topological group (X,T). $(X,F(T))$ is Q-C_1[3] if and only if (X,T) is C_1 .

Proposition 1.5. Let (X,J) be a fuzzy topological group which belongs to type (QL). Then the space of (X,J) is Hausdorff if it is T_0 .

Definition 1.2. A fuzzy topological space (X,J) is said to be Q-regular iff for any fuzzy point x_λ and any open Q--neighborhood U' Of x_λ there exists an open set V' such that $x_\lambda q \overline{V'} \subset U'$.

Proposition 1.6. The space of a induced fuzzy topological group $(X,F(T))$ is Q-regular.

Proof. To prove this proposition it is enough to verify that for any $\lambda \in (0,1]$ and any open Q-neighborhood U' of e_λ there exists a $V' \in F(T)$ such that $e_\lambda q \overline{V'} \subset U'$.

Let $\mathcal{U} = \{U\}$ be an open neighborhood base of e in (X,T). Then $\mathcal{U}_\lambda = \{(U \cap r_i^*) = U' \mid U \in \mathcal{U},\ r_i = \min(1, 1-\lambda+\frac{1}{i})\}$ is a Q--neighborhood base of e_λ in $(X,F(T))$. For any $U' = (U \cap r_i^*) \in \mathcal{U}_\lambda$, since $(X,F(T))$ is a fuzzy topological group we can find a $V' = (V \cap r_1^*)$ such that $V'V'^{-1} \subset U'$.

Suppose that $x_\mu \in \overline{V'} = (\overline{V \cap r^*})$. Then by the Theorem 4.1 in [3] there exists a fuzzy net S' in V' ,which converges to x_μ . Hence $\mu < r_1$. Furthermore we can find a $1 \geq r_s >$ $\max(\mu, 1-\mu)$ such that $x(V \cap r^*)$ is a Q-neighborhood of x_μ . Then $x(V \cap r^*)$ is quasi-coincident with $(V \cap r_1^*)$ at a point $y \in X$. Namely

$$\mu_{(V \cap r_1^*)}(y) + \mu_{x(V \cap r_s^*)}(y) > 1$$

Hence $y_{r_1} \in (V \cap r_1^*)$ and $(x^{-1}y)_{r_s} \in (V \cap r_s^*)$. Putting $r = \min(r_1, r_s)$. Now $x_r = y_r (x^{-1} y)_r^{-1} \in (V \cap r^*)(V \cap r^*)^{-1} \subset U'$. This implies that $x_\mu \in U'$ and then $\overline{V} \subset U'$. The proof thus is completed.

Proposition 1.7. Let $(X,F(T))$ be an induced fuzzy topological group on (X,T) and H be an ordinary subgroup of X . Then the coset space $(X/H,J^*)$ [4] of $(X,F(T))$ relative to H is regular.

Definition 1.3. A fuzzy net $S'=\{x^{(n)}_{\lambda_n}, n\in D\}$ in a fuzzy topological group (X,J) is called a λ-Cauchy net iff for any Q-neighborhood W' of e_λ we can find a $m\in D$ such that for any $n, n'\geqslant m$, $x^{(n)}_{\lambda_n}(x^{(n')})^{-1}_{\lambda_{n'}}$ is quasi-coincident with W' .

Proposition 1.8. A fuzzy net $S'=\{x^{(n)}_{\lambda_{n'}}, n\in D\}$ in an induced fuzzy topological group $(X,F(T))$ is a λ-Cauchy net iff the following conditions are satisfied .

(i) The ordinary net $S=\{x^{(n)}, n\in D\}$ in (X,T) is a Cauchy net.

(ii) For any $0<\varepsilon<\lambda$ there is a $m\in D$ such that for any $n\geqslant m$ there holds $\lambda_n>\lambda-\varepsilon$.

Definition 1.4. A fuzzy set A' in a fuzzy topological group (X,J) is said to be fuzzy complete iff any λ-Cauchy net in A' converges to a fuzzy point in A' .

Proposition 1.9. Let $(X,F(T))$ be the induced fuzzy topological group on (X,T). Then $(X,F(T))$ is fuzzy complete if and only if (X,T) is complete .

2. On the subgroups and quotient groups of an induced fuzzy topological group

Proposition 2.1. Let $(X,F(T))$ be the induced fuzzy topological group on (X,T), N be an ordinary subgroup of X and T_N be the relative topology of T on N and J_N be the relative fuzzy topology of J on N (where J=F(T)). Then the induced fuzzy topology $F(T_N)$ on (N,T_N) and J are equivalent.

Proof. Suppose that $\mathcal{U}=\{U\}$ is an open neighborhood base of e in (X,T) and $\lambda\in(0,1]$. Then by Proposition 1.2,

$\mathcal{U}_\lambda=\{U'=U\cap r_i^* \mid U\in\mathcal{U}, r_i=\min(1,1-\lambda+\frac{1}{i})\}$ is an open Q-neighborhood base of e_λ in (X,J). Now it is easy seen that $\mathcal{U}_\lambda^N=\{U'\cap N \mid U'\in\mathcal{U}_\lambda\}$ is an open Q-neighborhood base of e_λ in (N,J_N).

On the other hand since $\mathcal{U}_N=\{U\cap N \mid U\in\mathcal{U}\}$ is a neighborhood base of e in (N,T_N), it follows from Proposition 1.2,

$\mathcal{U}_N^\lambda=\{(U\cap N)\cap r_i^* \mid U\in\mathcal{U}, r_i=\min(1,1-\lambda+\frac{1}{i})\}$ is an open Q-neighborhood base of e_λ in $(N,F(T_N))$. Compareing $\mathcal{U}_\lambda^N$ with $\mathcal{U}_N^\lambda$ we can easily obtain the assertion of the proposition.

Proposition 2.2. Let (X,J) be an induced fuzzy topological group on (X,T) and N be an ordinary normal subgroup of X . Let (X/N,T*) be the quotient group of (X,T) relative to N and (X/N,J*) be the fuzzy quotient group of (X,J) relative to N. Then the induced fuzzy topology F(T*) on (X/N,T*) and J* are equivalent.

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