# Fuzzy Syntopogenous g–families and Fuzzy Syntopogenous Structures①

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Recently, the initial theory of fuzzy syntopogenous structures was established by A.K.Katsaras. In this paper, another structure—fuzzy. syntopogenous g–family is defined, it is studied that the relationship between fuzzy syntopogenous structrues and fuzzy syntopogenous g–families. Particularly, if $g(I_X)$is dense in L–fuzzy syntopogenous space $(Y,S_1)$,then $H_{S1} = \{G_{\ll_1} : \ll_1 \in S_1\}$ is an L–fuzzy syntopogenous g–family. Conversely, if H is an L–fuzzy g–family, then $S_{1H} = \{\ll_{1G} : G \in H\}$ and $S_{2H} = \{\ll_{2G} : G \in H\}$ are respectively L–fuzzy syntopogenous structures on Y and on X.Finally, it is discussed that fuzzy local syntopogenous structures and the image of fuzzy syntopogenous structures.

Keywords: Fuzzy Lattice, Fuzzy g–mapping, Fuzzy Syntopogenous g–family, Fuzzy Local Syntopogenous Structure, Image of Fuzzy Syntopogenous Structure.

## 1. Introduction

A.Csaszar [1] gave the concept of syntopogenous structures for the unified theory of topology, proximity and uniformity.A.K.Katsaras and C.G.Petalas[2,3,4] introduced the fuzzy syntopogenous structures and studied the unified theory of fuzzy topology, fuzzy proximity and fuzzy uniformity and obtained some similar properties. In this paper, another structure—fuzzy syntopogenous g–family is defined, it is studied that the relationship ybetween fuzzy syntopogenous structures and fuzzy syntopogenous g–families. Also it is discussed that fuzzy local syntopogenous structures and the image of fuzzy syntopogenous structures.

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## 2. Preliminaries

In this paper, $L = <L,\leq,\wedge,\vee,'>$ always denotes a completely distributive lattice with order–reversing involution "$'$"(i.e. fuzzy lattice). Let o be the least element and 1 be the greatest one in L.Suppose X is a nonempty(usual) set, an L–fuzzy set in X is a map A:X→L, and $L^X$will denote the family of all L–fuzzy sets in X.It is clear that $L^X = < L^X, \leqslant, \wedge, \vee, ' >$ is a fuzzy lattice, which has the least element $O_X$and the greatest one $1_X$, where $O_X(x) = 0, 1_X(x) = 1$for any $x \in X$.

The following prinicipal definitions and lemmas about fuzzy syntopogenous structures are similar to [2,3,4], they can be expanded to function domain L.

Definition2,1. A binary relation $\ll$on $L^X$is called an L–fuzzy semitopogenous order if it satisfies the following axioms:

(1) $O_X \ll O_X$ and $1_X \ll 1_X$;

(2) $A \ll B$ implies $A \leq B$;

(3) $A_1 \leq A \ll B \leq B_1$implies $A_1 \ll B_1$.

The complement of an L–fuzzy semi–topogenous order is the L–fuzzy semi–topogenous order $\ll^c$which is defined by $A \ll^c B$ iff $B' \ll A'$.

An L–fuzzy semi–topogenous order is called:

(a) symmetrical if $\ll = \ll^c$;

(b) topogenous if $A_1 \ll B_1$and $A_2 \ll B_2$ imply $A_1 \vee A_2 \ll B_1 \vee B_2$ and $A_1 \wedge A_2 \ll B_1 \wedge B_2$;

(c) Perfect if $A_j \ll B_j, j \in J$, implies $\vee A_j \ll \vee B_j$;

(d) biperfect if $A_j \ll B_j, j \in J$,implies $\vee A_j \ll \vee B_j$ and $\wedge A_j \ll \wedge B_j$.

Definition 2.2. an L–fuzzy syntopogenous structure on X is a nonempty set S of L–fuzzy topogenous orders on X having the following properties: ($LFS_1$) S is directed in the sense that given any two members $\ll_1$, $\ll_2$of S there exists $\ll$in S finer than both $\ll_1$and $\ll_2$, i.e.$\forall A,B \in L^X$,$A \ll_1 B$(or $A \ll_2 B$) implies $A \ll B$; ($LFS_2$) For each $\ll$in S there exists $\ll_1$in S such that $A \ll B$ implies the existence of an L–fuzzy set D with $A_1 \ll_1 D \ll_1 B$. The pair (X,S) is called an L–fuzzy syntopogenous space.

Lemma 2.1. Let S be an L–fuzzy syntopogenous structure on X, then the mapping $A \to A^0 = \vee \{B: B \ll A$, for some $\ll \in S\}$ is an interior operator and so it defines an L–fuzzy topology $T_1(S)$.If $\ll_S = \bigcup_{\ll \in S} \ll$,then $A \in T_1(S)$iff $A \ll_S^p A$.Conversely, for every L–fuzzy topology on X there exists a perfect L–fuzzy syntopogenous structure $S(T) = \{\ll\}$, where $A \ll B$ iff there exists $D \in T$ with $A \leq D \leq B$.

Proof.see [2].

### 3. L–fuzzy g–mappings and L–fuzzy semi–topogenous orders

Let us consider two nonempty usual sets X,Y and a single valued mapping $g:X\to Y$ (X,Y and g will be fixed throughout the whole paper)

Definition 3.1. A mapping $G:L^Y\to L(X)=\{K:\subseteq L^X\}$ is called an L–fuzzy g–mapping if it satisfies the following axioms:

(FMO) $G(A)\neq\Phi$, and $F>E\in G(A)$ implies $F\in G(A)$.

(FM1) $O_X\in G(A)$ iff $A=O_Y$.

(FM2) $E\in G(A)$ implies $g^{-1}(A)<E$.

(FM3) $A<B$ implies $G(B)\subset G(A)$.

G will be said to be topogenous. Further, if for any $E,F\in L^X$, and $A,B\in L^Y$, $E\in G(A)$, $F\in G(B)$ implies $E\wedge F\in G(A\wedge B)$, $E\vee F\in G(A\vee B)$.

Let G(X,Y) denote the set of all L–fuzzy g–mappings G defined on $L^Y$into L(X). A partial ordering"$\subset$" on G(X,Y) can be defined as follow

$G_1,G_2\in G(X,Y),G_1\subset G_2$ iff $G_1(A)\subset G_2(A)$,for any $A\in L^Y$.

$\{G_i:i\in I\}\subset G(X,Y),(\bigcup G_i)(A)=\bigcup G_i(A),(\bigcap G_i)(A)=\bigcap G_i(A)$.

It is easy that G(X,Y) is a complete lattice to the partial ordering "$\subset$" with the great element $G_1$and the least element $G_0$, where $G_1(A)=\{B:A\leqslant B,B\in L^X\}$ ,for any $A\in L^X$.

$G_0(O_X)=L^X,G_0(A)=\{1_X\}$ $for A\neq O_X$.

Definiton 3.2. Let ≪be an L–fuzzy semi–topogenous order [2], we call $X_0\subseteq X$ ≪ $-dense,if\ \vee L-fuzzy point\ x_\lambda\in L^X,u,v\in L^X,u$ ≪$v$,and $x_\lambda$L–quasie–coincident u, then $v\wedge 1_{X0}\neq O_{X0}$.

Theorem 3.1. If ≪ is an L–fuzzy semi–topogenous order on Y such that $g(1_X)$ is ≪–dense, then the definition $G_{\ll}(A)=\{E\in L^X:A\ll[g(E')]'\}$ yields an L–fuzzy g–mapping $G_{\ll}$which will be called the g–mapping induced from ≪. Further if ≪ is topogenous, then $G_{\ll}$is topogenous, too.

Before the proof of theorem 3.1, we give a lemma to explain theorem 3.1.

Lemma 3.2. Under the condition of theorem 3.1 $E\in G_{\ll}(A)$ iff there exists $E_0\in L^Y$such that A ≪$E_0$and $g^{-1}(E_0)<E$.

Proof. if $E\in G_{\ll}(A)$, choose $E_0=[g(E')]'$, then A ≪ $E_0$, $g^{-1}([g(E')]')=[g^{-1}(g(E'))]'\subset E''=E$.Conversely, if there exists $E_0\in L^Y$, $g^{-1}(E_0)\subset E$, A ≪$E_0$,then $E'\subset[g^{-1}(E_0)]'$. and $g(E')\subset g([g^{-1}(E_0)]')\subset E'_0$,further more $E_0\subset[g(E')]'$, A ≪$[g(E')]'$, $E\in G_{\ll}(A)$.

Proof of theorem 3.1 Let us consider the system of axioms defining an L–fuzzy semi–topogenous order (see def 2.1), and prove the validity of (FMO)–(FM3).

(FMO): A ≪$[g(1'_X)]'$, thus $1_X\in G_{\ll}(A)$. If $E\subset F$ and $E\in G_{\ll}(A)$, then by lemma 3.2 $F\in G_{\ll}(A)$.

(FM1): $O_Y$ ≪ $O_Y$implies $O_X\in G_{\ll}(O_Y)$. Conversely, $O_X\in G_{\ll}(A)$ implies A

$\ll [g[O'_X)]' = [g(1_X)]'$, simce $g(1_X)$ is nonempty usual set in Y, a usual point $x_1 \in [g(1_X)]'$ and $A(x) \neq 0$, it is easy that $x_1$ quasi-coincident A,as $[g(1_X)]' \wedge g(1_X) = O_Y$, but this contradicts the density of $g(1_X)$, so $A = O_Y$.

(FM2) If $E \in G_{\ll}(A)$ by lemma 3.2 there exists $E_0$ such that $A \ll E_0$ and $g^{-1}(E_0) \subset E$, so $g^{-1}(A) \subset E$.

(FM3) If $A \subset B$, $E \in G_{\ll}(B)$, then $B \ll [g(E')]'$ and $A \ll [g(E')]'$, so $E \in G_{\ll}(A)$. Topogenousity is omitted.

Further we shall consider an L-fuzzy semi-topogenous order on Y for any L-fuzzy g-mapping G.

Theorem 3.3. Let G be an L-fuzzy g-mapping, then an L-fuzzy semitopogenous order $\ll_{1G}$ can be defined on Y by the following formula:

$A \ll_{1G} B$ iff $A \subset B$ and $g^{-1}(B) \in G(A)$.

Further, if G is topogenous, then $\ll_{1G}$ is also topogenous.

Proof: Because of $g^{-1}(O_Y) = O_X$, $1_X \in G(1_Y)$ and $O_X \in G(O_Y)$, $1_X = g^{-1}(1_Y) \in G(1_Y)$, then $O_Y \ll_{1G} O_y$, $1_Y \ll_{1G} 1_Y$. If $A \ll_{1G} B$, by the definition, we have $A \subset B$, if $A_1 \subset A \ll_{1G} B \subset B_1$, obviously $A_1 \subset B_1$ and $g^{-1}(B) \in G(A)$ implies $g^{-1}(B_1) \supset g^{-1}(B) \in G(A) \subset G(A_1)$, hence $g^{-1}(B_1) \in G(A_1)$, $A_1 \ll_{1G} B_1$. By definiton2.1 $\ll_{1G}$ is an L-fuzzy semi-topogenous order. Topogenousity is omitted.

The L-fuzzy semi-topogenous order $\ll_{1G}$ was defined on Y.In another way one can determine an L-fuzzy semi-topogenous order $\ll_{2G}$ on X for an arbitrary L-fuzzy g-mapping G.

Theorem 3.4. If G is an L-fuzzy g-mapping, we have an L-fuzzy semi-topogenous order $\ll_{2G}$ on X given by the following definition: $A \ll_{2G} B$ iff $B \in G(g(A))$. Further, if G is topogenous, then $\ll_{2G}$ is also topogenous.

Proof: $O_X \ll_{2G} O_X$ and $1_X \ll_{2G} 1_X$ are obvious. If $A \ll_{2G} B$, then $B \in G(g(A))$, therefore by (FM2) $A \subset g^{-1}(g(A)) \subset B$. Finally suppose $A \subset A_1 \ll_{2G} B_1 \subset B$. Then $B \supset B_1 \in G(g(A_1))$, $g(A) \subset g(A_1)$, consequently because of (FMO) and (FM3) $B \in G(g(A))$, that is $A \ll_{2G} B$ holds. Topogenousity is omitted.

## 4. Fuzzy syntopogenous g-families and Fuzzy syntopogenous structures

Proposition 4.1. If $\ll_1$ and $\ll_2$ are L-fuzzy semi-topogenous orders on Y, $g(1_X)$ is $\ll_2$-dense, and $\ll_1 < \ll_2$, then $g(1_X)$ is also $\ll_1$-dense, and $G_{\ll_1} \subset G_{\ll_2}$, conversely, if $G_1$ and $G_2$ are L-fuzzy g-mappings and $G_1 \subset G_2$, *then* $\ll_{1G_1} \subset \ll_{1G_2}$ and $\ll_{2G_1} \subset \ll_{2G_2}$.

The proof is straight forward and hence omitted.

Theorem 4.2. Let G be an L-fuzzy g-mapping, we have an L-fuzzy g-mapping denoted by $G^2$ for which $E \in G^2(A)$ iff there exists $F \in G(A)$ such that $E \in G(g(F))$. $G^2 \subset G$, and it has the properties listed below:

(1). If $\ll$ is an L–fuzzy semi–topogenous order on Y, and $g(1_X)$ is $\ll$–dense, then $G_{\ll^2} \subset G^2$.

(2). If g is injective and G is topogenous, *then* $\ll_{1G^2} \subset \ll^2_{1G}$.

(3). $\ll_{2G^2} \subset \ll^2_{2G}$ always holds.

Proof: $G^2 \subset G$ is true, because $E \in G^2(A)$ implies the existence of an L–fuzzy set $F \in G(A)$ such that $E \in G(g(F))$, and by (FM2) $E \supset g^{-1}(g(F)) \supset F \in G(A)$, thus in view of (FMO) $E \in G(A)$. $G^2$ is an L–fuzzy g–mappping. (FMO):$1_X \in G^2(A)$, since $1_X \in G(A)$ *and* $1_X \in G(g(1_X))$. If $F \supset E \in G^2(A)$ then for a suitable $F_0 \in G(A)$, *we have* $F \supset E \in G(g(F_0))$,therefore $F \in G^2(A)$. (FM1): $O_X \in G(O_Y)$ *and* $O_X \in G(g(O_X))$, thus $O_X \in G^2(O_Y)$. Conversely, if $O_X \in G^2(A)$, *and* $F \in G(A)$ such that $O_X \in G(g(F))$, thus $F \subset g^{-1}(g(F)) \subset O_X$ and $F = O_X$, so $A = O_Y$. (FM2): $E \in G^2(A)$implies that there exists $F \in G(A)$ such that $E \in G(g(F))$, thus $g^{-1}(A) \subset F \subset g^{-1}(g(F)) \subset E$, then $g^{-1}(A) \subset E$. (FM3): $E \in G^2(B)$, and $A \subset B$ imply that there exists F such that $F \in G(B) \subset G(A)$ and $E \in G(g(F))$ then $E \in G^2(A)$.

(1). $E \in G_{\ll_1^2}$ (A) iff $A \ll_1^2 [g(E')]'$ iff there exists C such that $A \ll_1 C \ll_1 [g(E')]'$. Choose $F = g^{-1}(C)$,then $g(F) = g(g^{-1}(C)) \subset C, g(F) \ll_1 [g(E')]'$, i.e.$E \in G_{\ll_1}(g(F))$,*as* $C \subset [g(g^{-1}(C'))]'$. *So* $A \ll_1 [g(F')]'$, i.e. $F \in G_{\ll_1}(A)$.Thus $E \in G^2_{\ll_1}(A)$.

(2).$A \ll_{1G^2} B$iff $A \subset B, g^{-1}(B) \in G^2(A)$iff $A \subset B$, there exists $F \in G(A)$ such that $g^{-1}(B) \in G(g(F))$. Choose $C = A \vee g(F)$, then $A \subset C$, $g^{-1}(C) = g^{-1}(A) \vee g^{-1}(g(F))$, as g is injective, so $g^{-1}(g(F)) = F$, but $g^{-1}(B) \in G^2(A)$, we have $g^{-1}(B) \in G(A)$, and $g^{-1}(C) \in G(A)$, $g^{-1}(A) \subset F$.Also $g^{-1}(B) \in G(g(F))$ implies $F \subset g^{-1}(g(F)) \subset g^{-1}(B)$ and $C \subset B$.Because G is topogenous, $g^{-1}(B) \in G(A \vee g(F)) = G(C)$. Thus $A \ll_{1G} C \ll_{1G} B$ iff $A \ll^2_{1G} B$.

(3). $A \ll_{2G^2} B$ iff $B \in G^2(g(A))$ iff there exists $F \in G(g(A))$ such that $B \in G(g(F))$, i.e. $A \ll_{2G} F \ll_{2G} B$.

Definition 4.1. A family C of L–fuzzy topogenous g–mappings will be called an L–fuzzy syntopogenous g–family, if the following conditions are fulfilled:

($FC_1$) For any $G_1, G_2 \in C$ there exists $G \in C$ such that $G_1 \bigcup G_2 \subset G$.

($FC_2$) If $G \in C$ then there exists $G_1 \in C$ with $G \subset G_1^2$.

Theorem 4.3. If S is an L–fuzzy syntopogenous structure on Y such that $g(1_X)$ is S–dense (that is $g(1_X)$ is $\ll$–dense for every $\ll \in S$), then $C_S = \{G_\ll : \ll \in S\}$ is an L–fuzzy syntopogenous g–family. Conversely, let C be an arbitrary L–fuzzy syntopogenous g–family. then we have an L–fuzzy syntopogenous structure $S_{2C} = \{\ll_{2G} : G \in C\}$ on X.Further, if g is injective. Then $S_{1C} = \{\ll_{1G} : G \in C\}$ is an L–fuzzy syntopogenous structure on Y.

Proof: This theorem can be verified directly by theorem 4.1, 4.2.

## 5. L−fuzzy local syntopogenous structure

Definition 5.1. An L−fuzzy local syntopogenous structure on X is a nonempty set S of L−fuzzy topogenous order on X having the following properties:

($LFSO_1$) S is directed in the sense that given any two members $\ll_1$, $\ll_2$ of S there exists $\ll$ in S finer than both $\ll_1$ and $\ll_2$.

($LFSO_2$) For each $\ll$ in S there exists $\ll_1$ in S such that $X_\lambda \ll B$ implies the existence of an L−fuzzy set D with $x_\lambda \ll_1 D \ll_1 B$. The pair (X,S) is called an L−fuzzy local syntopogenous space.

Proposition 5.1. (1) If(X,S) is an L−fuzzy syntopogenous space, then (X,S) is an L−fuzzy local syntopogenous space. If $S^l = \{\bigcup_{\ll \in S} \ll\}$, $S^p = \{\ll^p : \ll \in S\}$, then $(X,S^l)$ and $(X,S^p)$ are respectively L−fuzzy local syntopogenous space perfect L−fuzzy syntopogenous space. If $Y \subset X$, then (Y, S|Y) is an L−fuzzy local syntopogenous space. (2) Let $f: X \to Y$ be a mapping. (Y,S) be an L−fuzzy local syntopogenous structure. Then $(X, f^{-1}(S))$ is an L−fuzzy local syntopogenous space. (3) If $\{S_i : i \in I\}$ is a family of L−fuzzy local syntopogenous structure on X, then $S = \bigvee_{i \in I} S_i$ is an L−fuzzy local syntopogenous structure on X. (4) If $\{(X_j, S_j) : j \in J\}$ is a family of L−fuzzy local syntopogenous space, suppose $X = \prod_{j \in J} X_j$ and $S = \prod_{j \in J} S_j$, then (X,S) is an L−fuzzy local syntopogenous space. (5). If (X,S) is a perfect L−fuzzy local syntopogenous space, then (X,S) is an L−fuzzy syntopogenous spqce.

The proof is omitted.

Proposition 5.2.Let (X,S) be an L−fuzzy syntopogenous space. For binary relation $<_s$ on X such that for any $x,y \in X$, $x <_S Y$ iff $A \in L^X$, $\ll \in S$, $\lambda \in L$, $\lambda \neq 0$, if $y_\lambda \ll A$ implies $x_\lambda < A$, then "$<_s$" is a preorder on X.

Proof: (1). Reflexive, since $x_\lambda \ll A$ implies $x_\lambda < A$; (2). transitive, if $X <_s y$, $y <_s z$ and for $A \in L^X$, $\ll \in S$, $\lambda \in L$, $\lambda \neq 0$, $Z_\lambda \ll A$, by ($LFSO_2$) there exists $\ll_1 \in S$ and $B \in L^X$, such that $z_\lambda \ll_1 B \ll_1 A$. As $y <_s z$, So $y_\lambda < B \ll_1 A$, i.e.$y_\lambda \ll_1 A$ also $x <_s y$, thus $x_\lambda < A$, i.e. $X <_s Z$.

## 6. Image of L−fuzzy syntopogenous space

Proposition6.1.Let $f: X \xrightarrow{onto} Y$ be a impping, $\ll$ be an L−fuzzy semitopogenous order on X, we define a binary relation $\ll_1$ on Y as follows: $A \ll_1 B$ iff $f^{-1}(A) \ll f^{-1}(B)$. *Then* $\ll_1$ is an L−fuzzy semi−topogenous order on Y, $\ll_1$ is called as the image of $\ll$ and denoted by f( ≪).

Proof: Because of $f^{-1}(O_Y)=O_X$ ≪$O_X=f^{-1}(O_Y)$and $f^{-1}(1_Y)=1_X$ ≪$1_X=f^{-1}(1_Y)$,$O_Y$ ≪$_1O_Y$and $1_Y$ ≪$_11_Y$,and$A$ ≪$_1B$implies $f^{-1}(A)$ ≪$f^{-1}(B)$,thus$f^{-1}(A) \leqslant f^{-1}(B)$, as for any $C \in L^Y$, $f(f^{-1}(C))=C$, so $f(f^{-1}(A)) < f(f^{-1}(B))$ i.e. $A<B$.Also $A_1 \leqslant A$ ≪$_1 B \leqslant B_1$ implies $f^{-1}(A_1) \leqslant f^{-1}(A)$ ≪$f^{-1}(B) \leqslant f^{-1}(B_1)$. So $f^{-1}(A_1)$ ≪$f^{-1}(B_1)$ i.e.$A_1$ ≪$_1B_1$, from above we get that ≪$_1$is an L-fuzzy semi-topogeous order on Y.

Proposition 6.2. Let F:$X \xrightarrow{onto} Y$be a mapping. (1) If ≪is an L-fuzzy topogenous (res.symmetrical, perfect, biperfect) order, then f( ≪ ) is an L-fuzzy topogenous (res.symmetrical, perfect, biperfect) order. (2) If ≪< ≪$_1$, then f( ≪)<f( ≪$_1$). (3).If ≪ is an L-fuzzy semi-topogenous order, then $f^{-1}$(f( ≪$_1$))< ≪$_1$, f( ≪$^c$)=f( ≪)$^c$,f( ≪)$^p$< f( ≪$^p$),f( ≪)$^b$< f( ≪$^b$) and f( ≪)$^q$< f( ≪$^q$).(4). If { ≪$_i$:i∈ I} is a family of L-fuzzy semi-topogenous orders on X, then $f(\bigcup_{i\in I}$ ≪$_i) = \bigcup_{i\in I} f($≪$_i)$and $f(\bigcap_{i\in I}$ ≪$_i) = \bigcup_{i\in I} f($≪$_i)$.

Proof: This proposition can be verified directlly.

Definition 6.1. Let f:$X \xrightarrow{onto} Y$be a mapping. S is an L-fuzzy syntopogenous structure on X, f is called compatible with S.If for any ≪∈S and A ≪B implies the existence of $C \in L^Y$ such that $A \leqslant f^{-1}(C) \leqslant B$.

Proposition 6.3. Let f be compatible with S, then $S_1=\{f($≪$)$: ≪$\in S\}$ is an L-fuzzy syntopogenous structure on $Y.S_1$ is called the image of an L-fuzzy syntopogenous structure and denoted by f(S).

The proof is straigt forward and hence omitted.

## REFERENCES


1. A.Császár. Foundations of general topology. Oxford: pergamon press.1963.
2. A.K.Katsaras On fuzzy syntopogenous structures. Rev. Roumaine Math. pures Appl. 30(1985) 419–431.
3. A.K.Katsaras and C.G.Petalas. On fuzzy syntopogence structures. SMAA 99(1984) 219–236.
4. Liu Wang-jin. Fuzzy proximity space redefined. Fuzzy sets and systems. 15(1985) 241–248.
5. Liu Wang-jin and Mo Zhi-wen. Connectedness on L-fuzzy syntopogenous space. Fuzzy sets and Systems. (to appear)