FUZZY RELIABILITY OF VOTING SYSTEMS

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ABSTRACT: This paper is one of continuations for <FUZZY RELIABILITY>. It provides the calculating method which can be used in calculation of a fuzzy reliability for the voting systems. In this paper author derives the series formulas by means of a basic conecepts and principles of the fuzzy reliability. This paper only considers the indexes of FA mode.

KEY WORDS: Fuzzy Reliability of the Voting Systems.

We still employ a classificatory method of a system for gemeral reliability theory. A voting system is defined as the system consisting of n($\geqq$3) elements such that system is successful when at least r ($\leqq$ n) elements are successful.

It is supposed that a voting system consist of n elements D_1, D_2, ..., D_n (as shown in figure), and it may be assumed that n elements are same, and failure of any element would occur independentyly of an operation of other components.

Now we employ the denotations as follows:

R - general reliability of a element.

R_s- general reliability of voting system.

λ - general failure rate of a element.

$\underset{\sim}{R}_s$- fuzzy reliability of voting system.

$\underset{\sim}{R}$ - fuzzy reliability of a element.

$\underset{\sim}{MTTF}_s$ - fuzzy mean life of voting system.

$\underset{\sim}{\lambda}$ - fuzzy failure rate of a element.

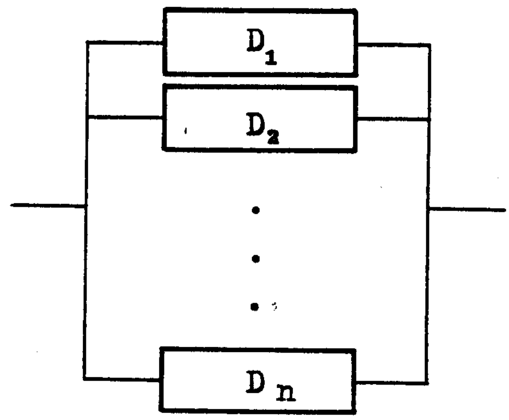
$\underset{\sim}{A}_i$ - discussing one of performance subsets.

$\mu_{\underset{\sim}{A}_i}(R_s)$ - degree of membership of R_s in $\underset{\sim}{A}_i$.

$\mu_{\underset{\sim}{A}_i}(R)$ - degree of membership of R in $\underset{\sim}{A}_i$.

D - the system is successful.

D_j - a element is successful.



(r - out - of - n)

Figure

By means of a definition of fuzzy conditional probability[3], we have

$$P(D \wedge \underset{\sim}{A}_i) = P(\underset{\sim}{A}_i | D)P(D) \quad (1)$$

$$P(D_j \wedge \underset{\sim}{A}_i) = P(\underset{\sim}{A}_i | D_j)P(D_j) \quad (2)$$

where the sign $\wedge$ denotes algebraic product.

By means of general reliability theory[2] and fuzzy reliability theory[1]

$$P(D) = R_s \quad , \quad P(D_j) = R$$

$$P(D \wedge \underset{\sim}{A}_i) = \underset{\sim}{R}_s \quad , \quad P(D_j \wedge \underset{\sim}{A}_i) = \underset{\sim}{R}$$

$$P(\underset{\sim}{A}_i | D) = \mu_{\underset{\sim}{A}_i}(R_s), \quad P(\underset{\sim}{A}_i | D_j) = \mu_{\underset{\sim}{A}_i}(R)$$

Therefore Eqs. (1) and (2) may be respectively rewritten as follows:

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}_i}(R_s)R_s \quad (3)$$

$$\underset{\sim}{R} = \mu_{\underset{\sim}{A}_i}(R)R \quad (4)$$

In terms of general probability theory, probability of k elements which are successful in n elements is

$$P_n^k = \binom{n}{k} R^k (1-R)^{n-k}$$

The system has (n-r+1) successful ways:

(1). k=r, r elements are successful and (n-r) elements are failure.

(2). k=r+1, (r+1) elements are successful and (n-r-1) elements are failure.

....

(n-r+1). k=r+(n-r) = n, every element is successful.

Then general reliability of voting system is

$$R_s = \sum_{k=r}^{n} \binom{n}{k} R^k (1-R)^{n-k} \quad (5)$$

By means of a relation between the general reliability and the general failure rate for every element[2], we have

$$R = \exp\{-\int_0^\infty \lambda dt\}$$

Substituting it into Eq.(5), we obtain

$$R_s = \sum_{k=r}^{n} \binom{n}{k} [\exp\{-\int_0^\infty \lambda dt\}]^k [1-\exp\{-\int_0^\infty \lambda dt\}]^{n-k} \quad (6)$$

If λ is equal to constant, then

$$R_s = \sum_{k=r}^{n} \binom{n}{k} [e^{-\lambda t}]^k [1-e^{-\lambda t}]^{n-k}$$

$$= \sum_{k=r}^{n} \binom{n}{k} e^{-k\lambda t} [1-e^{-\lambda t}]^{n-k} \quad (7)$$

Substituting Eqs.(4) and (7) into Eq.(3), we obtain

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}_i}(R_s) \sum_{k=r}^{n} \binom{n}{k} e^{-k\lambda t} [1-e^{-\lambda t}]^{n-k} \quad (8)$$

Eq.(8) is a general expression of fuzzy reliability of the system.

By means of a relation between general failure rate and fuzzy failure rate[1], we have

$$\underset{\sim}{\lambda} = \lambda - \frac{d\mu_{\underset{\sim}{A}_i}(R)}{\mu_{\underset{\sim}{A}_i}(R)dt} = \lambda - \overline{\mu'_{\underset{\sim}{A}_i}(R)} \quad (9)$$

where $\overline{\mu'_{\underset{\sim}{A}_i}(R)}$ is related derivative of $\mu_{\underset{\sim}{A}_i}(R)$.

Substituting Eq.(9) into Eq.(8), then

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}_i}(R_s)\sum_{k=r}^{n}\binom{n}{k}e^{-[\underset{\sim}{\lambda}+\overline{\mu'_{\underset{\sim}{A}_i}(R)}]kt}\cdot \{1-e^{-[\underset{\sim}{\lambda}+\overline{\mu'_{\underset{\sim}{A}_i}(R)}]t}\}^{n-k} \quad (10)$$

Now derive a formula of fuzzy mean life of voting system. By means of a relation between fuzzy mean life and fuzzy reliability,[1] $\underset{\sim}{MTTF}_s$ of the system is expressed

$$\begin{aligned}\underset{\sim}{MTTF}_s &= \int_0^\infty \underset{\sim}{R}_s dt \\ &= \int_0^\infty \mu_{\underset{\sim}{A}_i}(R_s)R_s dt \\ &\approx \mu_{\underset{\sim}{A}_i}(R_s)_m \int_0^\infty R_s dt \end{aligned} \quad (11)$$

where $\mu_{\underset{\sim}{A}_i}(R_s)_m$ is mean value of $\mu_{\underset{\sim}{A}_i}(R_s)$ which over interval $[0, \infty]$ of operational time t, it is equal to constant, and

$$\begin{aligned}\int_0^\infty R_s dt &= \int_0^\infty \sum_{k=r}^{n}\binom{n}{k}R^k(1-R)^{n-k}dt \\ &= \int_0^\infty \sum_{k=r}^{n}\binom{n}{k}e^{-k\lambda t}(1-e^{-\lambda t})^{n-k}dt \\ &= \frac{1}{n\lambda}+\frac{1}{(n-1)\lambda}+\frac{1}{(n-2)\lambda}+\cdots+\frac{1}{r\lambda} \\ &= \frac{1}{\lambda}\left(\frac{1}{n}+\frac{1}{n-1}+\frac{1}{n-2}+\cdots+\frac{1}{r}\right)\end{aligned} \quad (12)$$

Substituting Eqs.(9) and (12) into Eq.(11), we obtain

$$\underset{\sim}{MTTF}_s = \frac{\mu_{\underset{\sim}{A}_i}(R_s)_m}{\underset{\sim}{\lambda}+\overline{\mu'_{\underset{\sim}{A}_i}(R)}}\left[\frac{1}{n}+\frac{1}{n-1}+\frac{1}{n-2}+\cdots+\frac{1}{r}\right] \quad (13)$$

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