TRUTH-VALUED-FLOW INFERENCE

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Existent fuzzy reasoning models are almost built perhaps unconsciously even vaguely on the base of inference relation theory which is still lacking in a clear and deep analysis. To clarify this theory and resolve some puzzles occurred in existent situation, a rather serious analysis and statements on inference relation is given, and it is emphasized from the exposition that Mandani's model can not be explained on the base of inference relation theory very will. To find the rather reasonable base for it, a new framework of approximate reasoning --Truth-valued-flow Inference(TI) theory is presented in the main part of this paper.

keywords: inference channels base, truth valued flow inference, factors spaces

1. INTRODUCTION

Even though prominent progress has been occurred in the applied areas of approximate reasoning based on fuzzy sets theory , existent models of fuzzy reasoning are not perfect yet. The main difficulties exist in two points:

1) Theoretical defect of the very non-binary logic: the breaking of uniqueness of implication form occurs naturally in logic while it go out the gate of bi-values. there are many different formulae of implication able to pass the check from the respect of logic, they are all coincide with the Boolean implication whenever the truth values of anticedents and consequences of them return into {0,1}. We are lacking reason to receive or refuse which one of them.

How to adding theory in order to catch information of selecting appropriate implication form in concrete situation is the main task of non-binary logic. It is an important contribution of fuzzy logic that using fuzzy relation to describe implication brings us the possibility of expending the capacity of logical

information. It is possible ,not yet. Inference relation theory , the base of fuzzy reasoning,have to be investigated seriously. But as known, some puzzles even paradoxes exist.
For example, in spite of different kinds of implications' combination, such as'and','or', 'else' etc., the writing forms of inference rules is often written as

(1.1) if A_1 then B_1,......,if A_n then B_n

This form is often transferred into an inference relation R:

(1.2) $$R(x,y)=\bigvee_{i=1}^{n}\left(A_i(x)\wedge B_i(y)\right)$$

Even though it is useful in practical fuzzy control initiated by E.H.Mamdani, unfortunately,as the mention in Proposition 2.3 of this paper ,the correctness of this formula ,whatever the supports of A_i are cover the universe U or not, can not be proved seriously by existent inference relation theory.
2) Complexity of performing existent fuzzy reasoning models in practice, especially it becomes to a big problem when we want to realize fuzzy reasoning in hardware of computer.
We are trying to take a rather serious analysis on inference relation theory in This paper , using shadow-representation theory to adding the information of selecting . We devote to promoting this theory but do not be restrained in it. In order to overcome some practical difficult in constructing hard-ware systems of fuzzy reasoning, in order to give (1.2) a reasonable explanation,in order to unify fuzzy reasoning and other non-determinate reasoning, we present the Truth-valued-flow Inference method in this paper. TI is not restrained in inference relation framework but conserves closed relation to it.

2. CLARIFYING OF INFERENCE RELATION THEORY(BINARY CASE)

We have to first get a serious statement on binary inference relations.

DEFINITION 2.1 The binary inference relation of implication A--->B is that

(2.1) $$R_{A\to B}(x,y)=T(A(x)\to B(y))$$

where

(2.2) $$T(P\to Q)=\begin{cases}1, & T(P)=1,\ T(Q)=0;\\ 0, & else.\end{cases}$$

Obviously, we have that

(2.3) $$R_{A\to B} = A\times B + \bar{A}\times Y$$

DEFINITION 2.2 The first term and the second term of right side in (2.3) are called **real part** and **trivial part** of that inference relation respectively and denote that

$$R^{(1)}_{A\to B} = A\times B$$

$$R^{(2)}_{A\to B} = \bar{A}\times Y$$

DEFINITION 2.3 The combined relations of several implications are defined as follows:

(2.4) $$R_{(A_1\to B_1)\text{and}\ldots\text{and}(A_n\to B_n)} = \bigcap_{i=1}^{n} R_{A_i\to B_i}$$

(2.5) $$R_{(A_1\to B_1)\text{or}\ldots\text{or}(A_n\to B_n)} = \bigcup_{i=1}^{n} R_{A_i\to B_i}$$

(2.6) $$R^{(1)}_{(A_1\to B_1)\text{else.}(A_2\to B_2)} = R^{(1)}_{A_1\to B_1} + R^{(1)}_{\bar{A}_1 A_2\to B_2}$$

(2.7) $$R^{(2)}_{(A_1\to B_1)\text{else.}(A_2\to B_2)} = R^{(2)}_{A_1\to B_1} \cap R^{(2)}_{A_2\to B_2}$$

where the complementary of A is denoted by a bar and

$A_1 A_2 = A_1\cap A_2,\ A_1 + A_2 = A_1\cup A_2 (A_1\cap A_2 = \Phi)$.

PROPOSITION 2.1

(2.8) $$R^{(1)}_{(A_1\to B_2)\text{and}(A_2\to B_2)} = A_1\bar{A}_2\times B_1 + A_1 A_2\times B_1 B_2 + \bar{A}_1 A_2\times B_2$$

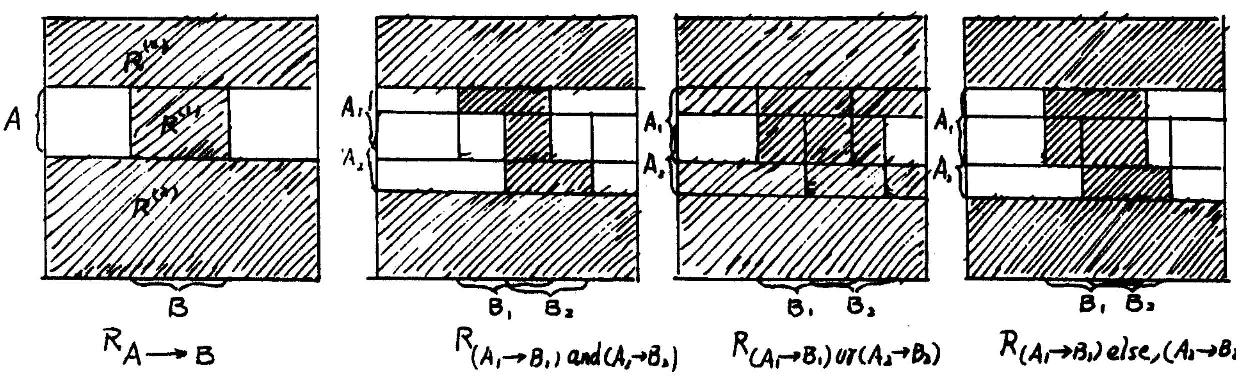
$$R^{(2)}_{(A_1\to B_1)\text{and}(A_2\to B_2)} = \overline{A_1\cup A_2}\times Y$$

(2.9) $$R^{(1)}_{(A_1\to B_1)\text{or}(A_2\to B_2)} = A_1 A_2\times (B_1\cup B_2)$$

$$R^{(2)}_{(A_1\to B_1)\text{or}(A_2\to B_2)} = \overline{A_1 A_2}\times Y$$

(2.10) $$R^{(1)}_{(A_1)\text{else.}(A_2\to B_2)} = A_1\times B_1 + \bar{A}_1 A_2\times B_2$$

$$R^{(2)}_{(A_1\to B_1)\text{else.}(A_2\to B_2)} = \overline{A_1\cup A_2}\times Y$$



We are interested in the special case of that $A_1=A_2+A$ or $B_1=B_2=B$. It is shown in the next proposition.

PROPOSITION 2.2

(2.11) $$R_{(A\to B_1)\text{and}(A\to B_2)} = R_{A\to B_1} \cap R_{A\to B_2}$$

(2.12) $$R_{A_1\to B)\text{and}(A_2\to B)} = R_{A_1\to B} \cup R_{A_2\to B}$$

(2.13) $$R_{(A\to B_1)\text{or}(A\to B_2)} = R_{A\to B_1} \cup R_{A\to B_2}$$

(2.14) $$R_{(A_1\to B)\text{or}(A_2\to B)} = R_{A_1\to B} \cap R_{A_2\to B}$$

(2.15) $$R_{(A\to B_1)\text{else},(A\to B_2)} = R_{A\to B_1}$$

(2.16) $$R_{(A_1\to B)\text{else},(A_2\to B)} = R_{A_1\to B} \cup R_{A_2\to B}$$

PROPOSITION 2.3 The necessary and sufficient condition of that

(2.17) $$R_{(A_1\to B_1)\text{and}\dots\text{and}(A_n\to B_n)} = R_{(A_1\to B_1)\text{or}\dots\text{or}(A_n\to B_n)}$$

$$= R_{(A_1\to B_1)\text{else},\dots\text{else}(A_n\to B_n)} = \bigcup_{i=1}^{n} (A_i \times B_i)$$

is that

$$A_1 + A_2 + \dots + A_n = X, \quad A_i \neq A_j (i \neq j)$$

Those propositions tell us that the combinations of implications have to be indicated in the forms of (2.4)--(2.7), generally they are different each other, and their inference relation are different with (1.2) except that A1,...,An form a division of X.

3. DESCRIBING IMPLICATIONS BY MEANS OF FACTOR SPACE THEORY

To find out an new approach for explain Mamdani's formula, we have to think of the meaning and representation of implications. Implication is a non-defined concept in logic, although logician are absorbed in describing the concept of implication by means of the contained relation of sets, The main relationship is that implication 'if x is P then x is Q' holding true is equivalent to:

$$P(\text{extension of concept } P) \subseteq Q(\text{extension of concept } Q).$$

When the antecedent and consequence of an implication are able to be described in a same universe of discussion, the meaning of implication can be described by 'contained ' relation. Unfortunately, their universes are different in general. But in our opining implication is a relation between propositions which reflects causality of them. Even though the antecedent and reflects causality of them. Even though the antecedent and consequence of an implication are described in different universes, they should be found in a common factors space and occurred some contained relation in that space caused by the causality, Let us restate the definitions of factors spaces.

DEFINITION ([8]) A **factors space** is a family of sets { X_f }($f \in F$) with a Boolean algebra F as its index set and satisfies that

1) $X_0 = \Phi$;

2) $f \wedge g = 0$ *impliesthat*

$$X_{f \vee g} = X_f \times X_g$$

where $F = F(\vee, \wedge, c, 1, 0)$

f in F is called factor, X_f is called states (or characteristic, phase) space of f, X_{f^c} is called complementary space of X_f, X_1 is called wh le space.
Roughly speaking, a factor space is a family of states (or characteristic, phase) spaces being a familiar term in control (recognition,physics) theory, but Factor space theory emphasizes the varying of states spaces with the varying of factors.

DEFINITION ([8]) Let O be the universe of objects concerned with a family of concepts which can be represented as fuzzy subsets of O. Mapping r : O$\longrightarrow$ X_1 is called representation of O and r(A) is called the representation of concept A in F(O)). Denoted

(3.1) $$\tau(A) = \bigwedge\{f \mid f \in F, \uparrow^1(\downarrow_f(r(A))) = r(A)\}$$

which is called **rank of concept** A, where $\downarrow_f$ denotes project to X_f, $\uparrow^1$ denotes cylindric extends to X_1.

DEFINITION 3.1 Let { X_f } ($f \in F$) be a factors space, $P \in F(O_1)$, $Q \in F(O_2)$, $r_i:O_i \longrightarrow X_i$ be representation of $O_i (i=1,2)$. We call that P implies Q denoted $P \longrightarrow Q$ if they satisfy that

(3.2) $$\uparrow^{\tau(Q)}(\downarrow_h r_1(P)) \subseteq \downarrow_h r_2(Q)$$

PROPOSITION 3.1

(3.3) $$p \to q, p' \subseteq p, q'q \Rightarrow P' \to Q'$$

(3.4) $$P \to Q, P' \subseteq\to Q' \Rightarrow P \vee' \to Q \vee Q', P \wedge P' \to Q \wedge Q'$$

4. INFERENCE CHANNELS AND THEIR BASES

Viewing the inference process as truth values flow along the channels of linking antecedent to consequence of implications, we have to give some axioms on inference channels according to the properties of implications mentioned above.

DEFINITION 4.1 Let C be a subset of F (O_1)X $F(O_2)$, we call a set of inference channels under a given knowledge if

(4.1) $1)(\Phi,Q),(P,O_2)\in C$;

(4.2) $2)(P,Q)\in C,P'\subseteq P,Q'Q\Rightarrow(P',Q')\in C$;

(4.3) $3)(P_1,Q_2),(P_2,Q_2)\in C\Rightarrow(P_1\vee P_2,Q_1\vee Q_2),(P_1\wedge P_2,Q_1\wedge Q_2)\in C$

channel (P,Q) can be written as $P \longrightarrow Q$.
The meaning of axiom 2) is clear: the smaller set at head and the bigger set at the tail, the weaker implication. So it is necessary to define a relation of representing the value of information of channels.

DEFINITION 4.2 Denote

(4.4) $\succ = \{(P_1\to Q_1,P_2\to Q_2)\mid P_i\to Q_i\in C(i=1,2);P_1\supseteq P_2,Q_1\subseteq Q_2\}$

it is called **validity relation**, we call channel $P_1 \longrightarrow q_1$ is more valuable than $P_2 \longrightarrow Q_2$ if (P1,Q1) $\succ$(P2,Q2). (P,Q) is called a valuable channel if there is no (p',Q')$\succ$(P,Q).

PROPOSITION 4.1 $(C,\succ)$ forms a poset, and (C^*, $\vee$, $\wedge$) forms a lattice where C^* is the set of valuable channels and

$$(P_1,Q_1)\vee(P_2,Q_2)=(P_1\vee P_2,Q_1\vee Q_2)$$

$$(P_1,Q_1)\wedge(P_2,Q_2)=(P_1\wedge P_2,Q_1\wedge Q_2)$$

DEFINITION 4.3 We call a subset B of C^* a **base** of C^* if C^* is the smallest closure of B under $\vee$ and $\wedge$,i.e. $C^* = [B]_{\vee\wedge}$. We call B a **left-ward base** if it is a base and

$$.B=\{P\mid \exists Q:(P,Q)\in B\}$$

forms a linguistic division of O_2, i.e., .B is the set of linguistic values of a linguistic variable. We call E={({u},Q) | u in Q_1} a **point-base** of C if $C^* = ([E]_{\vee,\wedge})^*$.

5. TRUTH-VALUED- FLOW INFERENCE

Illustratively understanding by the name of TI, we can imagine the inference process as the flows of truth values along the channels of linking the anticedent to consequence of implication concerned.

Step 1 (pretreatment) the knowledge being used to inference provides us a base of inference channels B. It prefer a left-ward base held the form as fallows

(5.1) $$P_{i1} \cup \ldots \cup P_{im_i} \longrightarrow Q_i \quad (i = 1, \ldots, n)$$

The fact P' is viewed as a generator of truth values, which get truth value at any P_{ij} as follows:

(5.2) $$\lambda_i = near(P_{ij}, P') = \bigvee_{u \in U} (P_{ij}(u) \wedge P'(u))$$

Step 2 (truth values flow) Put the heads of each channel of B on the generator P and get truth values λ_i = near(P_i,P). Each channel transfers λ_i from its head to its tail respectively, and get the consequence as that

(5.3) $$T(Q_i) = \lambda_i \quad (i = 1, \ldots, n)$$

here, if the channel is not simple but hold the form as (5.1) then we need next principle.

V-PRINCIPLE Let

$$(P,Q) = \left(\bigcup_{j=1}^{m} P_j, Q \right) = \bigvee_{j=1}^{m} (P_j, Q)$$

be a complex channel combined from several channels, the truth value inputed into its head is the maximal of truth values inputed into the heads of each simple channel involved.

Step 3 (truth value convertor) There many ways, for example, taking combination

$$Q'(v) = \bigvee_{i=1}^{n}{}^{*} (\lambda_i \wedge^{*} Q_i(v))$$

where $\vee^*$ and $\wedge^*$ be the pair of generalized fuzzy operators, and then the convertor can be taken in any way. We get determinate value even directly from (5.3).

6. FUZZY INFERENCE RELATION THEORY

Continuing the section 2, we give a serious analysis on the fuzzy inference relation based on inference channel's analysis and applied the shadow -representation theory.

Giving a simple information: 'if u is P then v is Q', When P,Q are both ordinary subsets, the point base of C determined by the information can be found and the graph of it is the relation (2.3). When P,Q are fuzzy subsets, how do we do?

According to the theory of shadow-representation, for a given fuzzy subset P on U, there is a class of random sets defined on

$$(\Omega, F, p; U, D, \hat{D})$$

(see [9]) such that their covering function equal to P:

(6.1) $$\mu_{\xi}(u) = p(\omega \mid \xi(\omega) \ni u) = P(u)$$

where   is one of random sets in that class, which is a mapping

$$\xi : \Omega \to D, \quad F - \hat{D} \text{ measurable}$$

To determine a random set corresponding to a fuzzy subset, there must define selections. So call a **selection** s, it is a mapping

$$s : F(U) \to \Omega^{D}$$

(6.2) $$s(P) : F - \hat{D} \text{ measurable} : \mu_{s(P)} = P \quad (P \in F(U))$$

DEFINITION 6.1 Giving a fuzzy implication P->Q, P,Q are fuzzy subsets on U,V resp., suppose that P,Q can be represented as fuzzy shadows of random sets defined on

$$(\Omega, F, p; U, D_1, \hat{D}_1) \wedge (\Omega, F, p; V, D_2, \hat{D}_2)$$

resp., giving the selections s_1 and s_2 resp., the fuzzy inference relation of implication P->Q is defined as follows:

(6.3) $$R_{P \to Q}(u,v) = p\left(\omega \mid R_{s_1(P)(\omega) \to s_2(Q)(\omega)} \ni (u,v)\right)$$

note that the inference relation occurred in the brackets is an ordinary inference relation defined in (2.3).

Different selections determine different fuzzy inference relations, the public selection is cut- selection. For a given fuzzy subset Q on U, the **cut-selection** is defined as follows:

$$s^* = s^*_Q = c o \lambda$$

where $\lambda : \Omega \to [0,1]$ *is a random variable uniformly distributed* $\in [0,1]$

$$c : [0,1] - \to P(U) : \quad \lambda(Q) = Q_\lambda = \{u \in U \mid Q(u) \geq \lambda\}$$

PROPOSITION 6.1 (cut-inference relation) under cut-selections, the fuzzy inference relation is that

(6.4) $$R_{P \to Q}(u,v) = 1 - m[P(u) \wedge Q(v), P(u)]$$

where [a,b] is interval and m is the Lebesgue measure.

PROPOSITION 6.2 (cut-combined inference relation) Under cut-selections, we have that

(6.5) $$R_{(P_1 \to Q_1) \text{and} \ldots \text{and} (P_n \to Q_n)} = 1 - m\left(\bigcup_{i=1}^{n} [Q_i(v) \wedge P_i(u), P_i(u)]\right)$$

(6.6) $$R_{(P_1 \to q_1) \text{or} \ldots \text{or} (P_n \to q_n)} = 1 - m\left(\bigcap_{i=1}^{n} [q_i(v) \wedge P_i(u), P_i(u)]\right)$$

(6.7) $$R_{(p_1\to Q_1)else,\dots else(p_n\to Q_n)} = 1-m\left(\bigcup_{i=1}^{n}[Q_i(v)\vee\left(\bigvee_{j=1}^{i-1}P_j(u)\right)\wedge P_i(u),P_i(u)]\right)$$

NOTE 1. For mula (6.4) is coincide with Lukasiewicz-Zadeh's inference relation , indeed we have that

$$1-mP(u)\wedge Q(v),P(u)=(1-P(u)+Q(v))\wedge 1=P(u)\dot{+}Q(v)$$

NOTE 2. From Proposition 6.2 we can see that Mamdani's formula (1.2) can not be explained by the inference combined relation, but it can be explained by means of truth-vàlued -flow inference because, the output of inference process according to Mandani's formula is that

$$Q'(v)=(P'oR)(v)=\bigvee_{u\in U}\left(P'(u)\wedge\bigvee_{i=1}^{n}\left(P_i(u)\wedge Q_i(v)\right)\right)$$

$$=\bigvee_{i=1}^{n}(\left(\bigvee_{u\in U}P'(u)\wedge P_i(u)\right)\wedge Q_i(v)$$

$$=\bigvee_{i=1}^{n}(near(P',P)\wedge Q(v))=\bigvee_{i=1}^{n}\left(\lambda_i\wedge Q_i(v)\right)$$

This is coincide with (5.4). Truth-valued-flow inference model is able to explain the Mamdani's formula, which has the convenience that we do not must thinking a group of inference channels has to be represented as a combined inference relation, we can respect these channels perform their function independently: transfers the truth value from their heads to their tails respectively. Of course, we can get a whole consequence in the step of truth valued convertor.

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