THE COMPLEX FUZZY METRIC SPACE

---THE GREY METRIC SPACE

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I. FOREWORD

Interval-type and information-type grey numbers are mostly used in practice The definitions of two kinds of grey numbers have been in article[2] [3] as follows:

$$u(x)=\begin{cases}\{1\}, & x\in[a,b]\\ \{0\}, & x\notin[a,b]\end{cases}\quad a,b\in R\ ,\ a\leq b$$

We called those grey numbers that are written in the above pattern interval-type grey numbers . And the interval-type grey numbers can be written as [a,b], in which a , b are respectively called left , right endpoint of the interval-type grey number u(x) , of which infu(x)=1 , supu(X)=1 . And the described interval grey numbers are the very hieravchy-type grey numbers which were advanced by Prof.Deng.

AS follows:

$$u(x)=\begin{cases}\{0\ ,1\ \} & x\in[a,b]\\ \{\ 0\ \} & x\notin[a,b]\end{cases}\quad a,b\in R\quad a\leq b$$

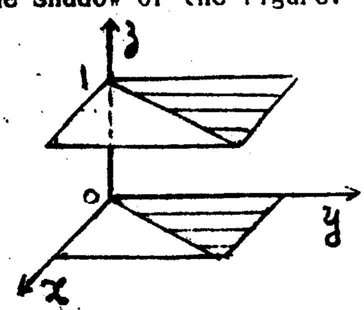
we called those grey numbers which can be written as the above pattern information-type grey number , which can be written as $[\underset{\sim}{a,b}]$,in which a, b are respectively called left ,right endpoint of information-type grey number u(x),of which infu(x)=0 ,supu(x)=1 . They are also called Deng's grey number.

Information-type, interval-type grey numbers are called by a joint name classical rational grey number . It is obvious that the classical rational grey number becomes fuzzy number when it is interval-type grey number.Therefore ,classical rational grey numbers are also called complex fuzzy numbers.

Grey (or complex fuzzy) distance is often given out in the course of studing these states of grey system so that concept of " The arbitrary approach " can be described exactly . Therefore, it's necessary to set up a grey distance and discass its basic properties on the bases of classical rational grey numbers.

II. GREY POINT AND THE GREY DISTANCE

We can know from its definition that any classical rational grey number u(x) is one-to-one correspondence with three ordered real numbers --left endplont a , its right endplont b and its infimum c . The three ordered real numbers determine only a point G in spatial rectangutal coordinates system. The point G is called a grey point of classical rational grey number (or a complex fuzzy point). It is easy to know that the set of classical rational grey number is one-to-one correspondence with the grey point set $\{(x,y,z) | x,y \in R,\ X \leq Y\ \ Z \in \{0,1\}\}$.The set of grey point can be expressed as two half plane as the shadow of the figure.



We give out the definition of grey (or complex fuzzy) distance as follows:

Let $u_1(x)$, $u_2(x)$ are two arbitrary classial rational grey numbers.

We define the distance between them as folows:

$D[\ u_1(x),\ u_2(x)\ ]=\max\{|a_2-a_1|,|b_2-b_1|,|\inf u_2(x)-\inf u_1(X)|\}$ where a_1, a_2 and b_1,$b_2$respectively the left ,right endplont of $u_1(X)$ and $u_2(X)$.

III. PROPERTIES OF THE GREY DISTANCE

property 1 . As for two arbitrary classical rational grey numbers $u_1(X)$, $u_2(X)$ there is

$$D[u_1(X), u_2(x)]=D[u_2(X), u_1(X)]$$

property 2. Let $u_1(X)$, $u_2(X)$ are two classical rational grey numbers , then $D[u_1(X), u_2(x)]=0$ if and only if $u_1(X)=u_2(X)$.

property 3. As for three arbitrary classical rational grey numbers $u_1(x)$, $u_2(x)$,$u_3(x)$ there is

$$D[u_1(x),u_2(x)]\leqslant D[u_1(X), u_3(x)]+D[u_2(x),u_3(x)]$$

proof: AS $D[u_1(x),u_2(x)]=D[u_2(x),u_1(x)]$ is true.It is proved under the following seven separate conditions

When $u_1(x)=[a_1,b_1]$, $u_2(x)=[a_2,b_2]$, $u_3(x)=[a_3,b_3]$

or $u_1(x)=[a_1,b_1]$, $u_2(x)=[a_2,b_2]$, $u_3(x)=[a_3\underset{\sim}{,}b_3]$

or $u_1(x)=[a_1\underset{\sim}{,}b_1]$, $u_2(X)=[a_2\underset{\sim}{,}b_2]$, $u_3(x)=[a_3,b_3]$

or $u_1(x)=[a_1\underset{\sim}{,}b_1]$, $u_2(X)=[a_2\underset{\sim}{,}b_2]$, $u_3(X)=[a_3\underset{\sim}{,}b_3]$

there is $D[u_1(x) , u_2(x)]=\max\{|a_2-a_1|,|b_2-b_1|,0\}=\max\{|a_2-a_1|,|b_2-b_1|\}$

When $D[u_1(x),u_2(x)]=|a_2-a_1|$ there is

$$D[u_1(X),u_2(X)]=|a_2-a_1|=|a_2-a_3+a_3-a_1|\leqslant|a_2-a_3|+|a_3-a_1|$$

$$\leqslant D[u_1(X),u_3(x)]+D[u_2(x),u_3(x)]$$

same method can proof:

When $D[u_1(X), u_2(X)]=|b_2-b_1|$

$$D[u_1(x), u_2(X)]\leqslant D[u_1(X), u_3(X)]+D[u_2(x), u_3(x)].$$

Therefore property 3 is true under the above four conditions.

When $u_1(x)=[a_1,b_1]$, $u_2(x)=[a_2\underset{\sim}{,}b_2]$, $u_3(x)=[a_3,b_3]$

or $u_1(X)=[a_1\underset{\sim}{,}b_1]$, $u_2(X)=[a_2,b_2]$, $u_3(X)=[a_3,b_3]$

or $u_1(X)=[a_1\underset{\sim}{,}b_1]$, $u_2(X)=[a_2,b_2]$, $u_3(x)=[a_3\underset{\sim}{,}b_3]$

there is $D[u_1(X), u_2(X)]\leqslant D[u_1(X), u_3(X)]+D[u_2(X), u_3(X)]$

(to be proved with similar method).

Therefore property 3 is true.

Because the grey distance which we define possesses the three properties of the metric space ,we define a metric space which is called grey (or complex fuzzy) metric space.

IV、THE RELATIONS BETWEEN GREY DISTANCE AND TWO REAL NUMBER'S RETANCE

The classical rational grey number is an extension of real number. And the real numbers are one part of classical rational grey numbers . We will then give out a theorem so as to illustrate that grey distance is an extension of distance of two real numbers.

Theorem: If $u_1(x)$, $u_2(x)$ are all rael numbers,then

$$D[\,u_1(x),\ u_2(x)]=|\,u_2(x)-\,u_1(x)|.$$

Proof: since $u_1(x)$, $u_2(X)$ are all real numbers

hence $u_1(x)=[a\ ,a\ ]$, $u_2(x)=[b\ ,b\ ]$

$$D[\,u_1(x),\ u_2(x)]=\max(|b-a|,|b-a|,0)=|b-a|=|\,u_2(X)-\,u_1(x)|$$

$$D[\,u_1(x),\ u_2(x)]=|\,u_2(x)-\,u_1(x)|$$

Reference

[1] Deng Ju Long , The grey controlling system .
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[2] Wang Qing Qin Wu He Qin ,The Theory of compound fuzzy sets, proceeding of NAFIPS' 88

[3] Wang Qin Yin Wu He Qin The concept of grey number and its property , Proceeding of NAFIPS'88